EXPLORATIONS WITH ROTATIONAL SOLIDS

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ABSTRACT

The paper deals with the importance of involving junior high school students in exploratory activities related to rotational solids. A pilot experiment carried out with 5-6 graders is discussed in terms of variety of problems solved by means of appropriate computer applications. Some ideas about the educational value of the learning-by-doing-and-creating approach are illustrated (also in the context of rotational solids) during an in-service teacher training course on IT.

INTRODUCTION

The importance of introducing an inquiry based style in the mathematics and science classes has been addressed by educators and policy makers including in recent reports of the European Commission (cf. [1]).

In this paper we are focusing on the experience gained when applying such a style in the context of rotational solids.

In the traditional mathematics curriculum for the 6th grade of Bulgarian schools, 3D topics include straight circular cylinder, straight circular cone and sphere. Ways for obtaining them via rotation of a rectangle, a rectangular triangle and a circle are shown (mainly by paper models) together with the formulae for the surface and the volume of these solids.

Some ideas for organizing the exploratory activities related to discovering patterns in the context of rotational solids have been presented in [2], [3]. When involved in activities dealing with discovering patterns, the students are expected to solve more easily a whole class of problems, to develop their mathematical intuition, and to get a better sense of some basic facts referring to the relation between the measurements in a specific space.

The applications our students used for studying various properties of rotational solids via virtual experiments were the *Elica Dalest* computer applications (*Potter's Wheel*, *Math Wheel* and *Bottle Design*) [4]. Their typical features include an easy start combined with a great potential for explorations. From a didactical point of view, the difference is determined mainly by the final objective of the scenario – to model an object from the real world by means of sufficiently close approximation, or to compute the volume and the surface of solids that are a combination of the simplest ones.

These specifics will be better demonstrated in the context of the problems presented to and solved by the students.

PLAYING, CONJECTURING AND VERIFYING WITH *POTTER'S WHEEL*

When working with the program, the students:

- used different figures to be rotated - a segment, a triangle, a square, a circle, a sine fragment, a *free* curve (Figure 1.); then they explored various positions of the rotated object (Figure 2.) in a rather chaotic manner and eventually – verified their conjectures about the solid to be obtained.

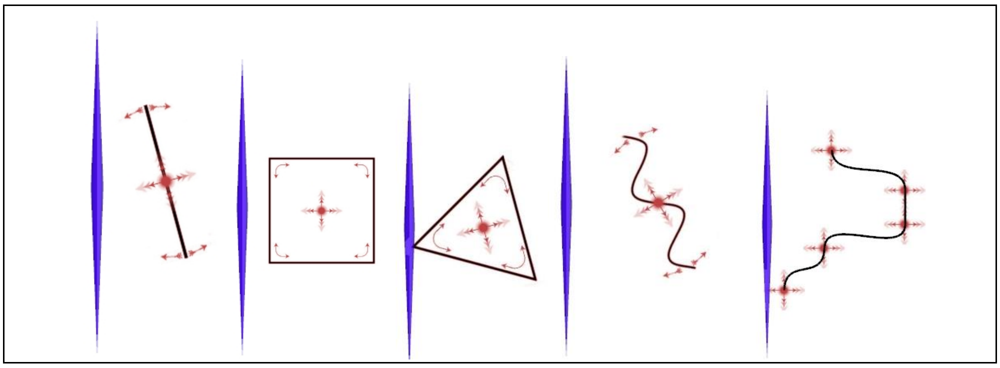


Figure 1. The *Math Wheel* objects under rotation

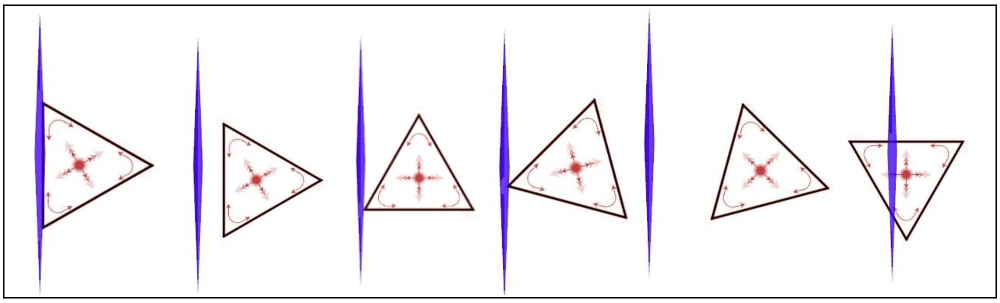


Figure 2. Exploring the effect of different positions of the rotated triangle

- used the options for showing consecutively the contour, the rotational solid and the cross-section (Figure 3.).

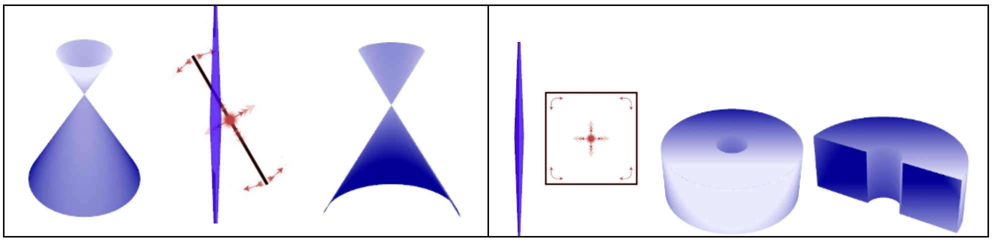


Figure 3. The objects under rotation, the solids they generate and their cross-sections.

- observed the solids from different perspectives (Figure 4.);

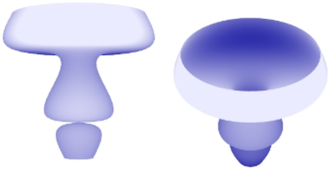


Figure 4. A rotational solid from different perspectives

- modeled two objects chosen among the ones shown in Figure 5.

Figure 5. A collection of real-world objects used for modeling with *Math Wheel*

Here are some of the computer models of objects chosen by the kids (Figure 6.):

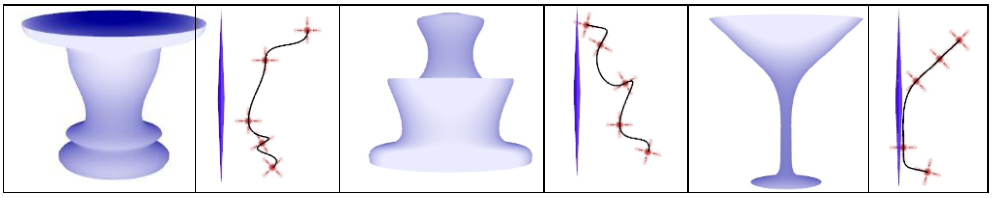


Figure 6. Computer models of the chosen real-world objects.

Those who chose to model the glass shared: *This one is the easiest to do. I'll make it in no time…*

However, at the moment the teacher posed a problem in a contest-like form (*Who will get the best model of the hat for a fixed time?),* all the participants demonstrated a high ability to concentrate and achieved good results.

When playing with *Potter's Wheel* the students explored a rich variety of rotational objects for a relatively short time by means of generating figures, different perspectives, etc. The problems posed directed their thinking to figuring out how to find the volume of more complex solids (e.g. by adding or removing certain parts similarly to what they had done with 2D figures). The goals of the activities included modeling and transformation of a rotational solid by given parameters – a contour, intersections, volume, etc. In addition, the rotational solids obtained were a good source for artistic ideas.

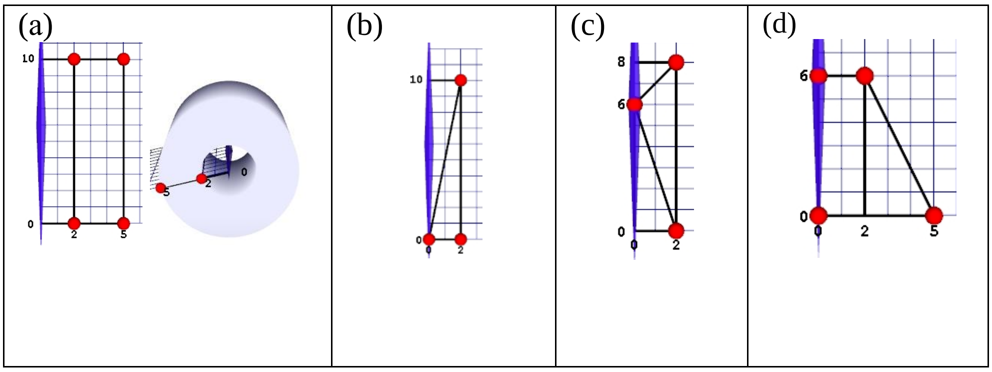
In view of the user-friendliness of the program, it was possible to acquire the technical details during the process of problem solving.

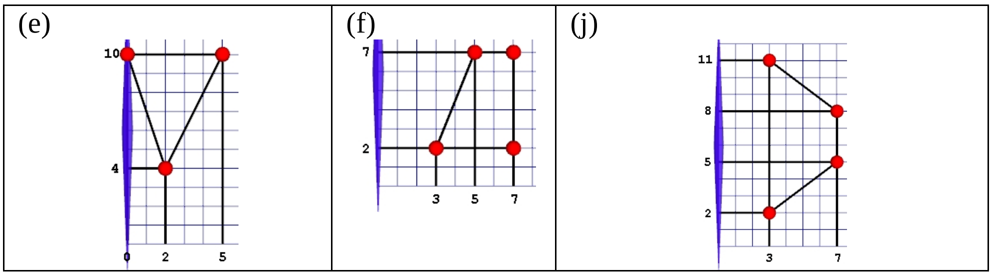
COMPUTING THE VOLUME OF ROTATIONAL SOLIDS

A specific feature of *Math Wheel* and *Bottle Design* is that together with the moving points which form the rotating figure, these programs provide a grid of square units to facilitate the measurement of the needed quantities.

Below we present a system of problems on computing the volume of rotational solids we offered to our pilot students.

Problem 1. *Find the volume of the rotational solids generated by Math Wheel*:





The rotational solids could be considered as a difference of the volumes of two cylinders (a), a cylinder of which a cone has been cut out (b) and as a cylinder of which two cones have been cut out (c).

For the case (c), some of the students needed help with the shape of the rotational solid whose volume they were expected to find. The reason was that they had not figured out that the object under rotation was the triangle formed by the three movable points rather than the polygon passing through them (as it is within *Bottle Design*) (Figure 7.).

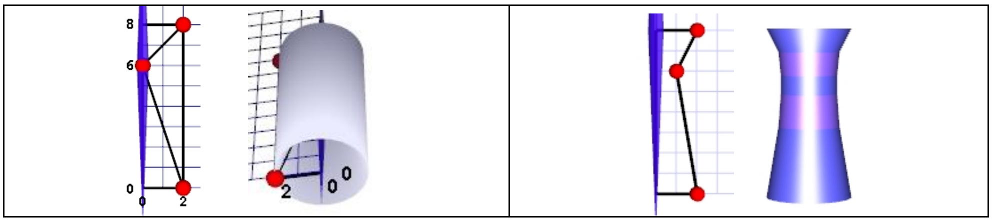


Figure 7. The difference when rotating a triangle *(Math Wheel*) and a polygon (*Bottle Design*).

Thus, they would find the volume of the *sand clock*, i.e. the solid complementing the real one to a cylinder.

The notion of *truncated cone* has dropped out of the curriculum in the recent years for good reasons.

During the after session discussion with the students, it became clear that this problem turned out to be the most difficult for them. This was to be expected since they had to trigger their thinking from *dividing a solid in easy-to-measure parts* to *complementing a given solid with another one to get a familiar solid*. Thus, in order to enhance the flexibility of their thinking, we had to encourage them to observe the given solid more carefully and to take their time (Figure 8.).

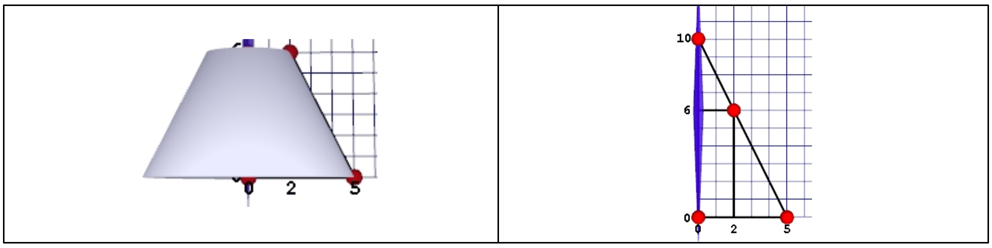
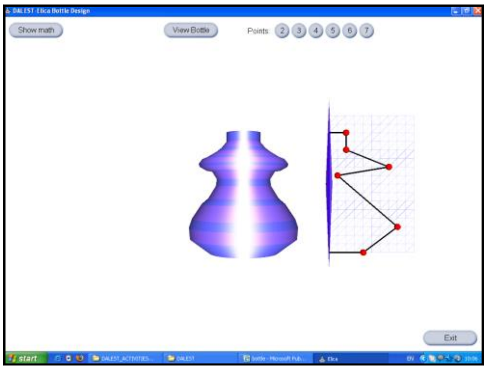


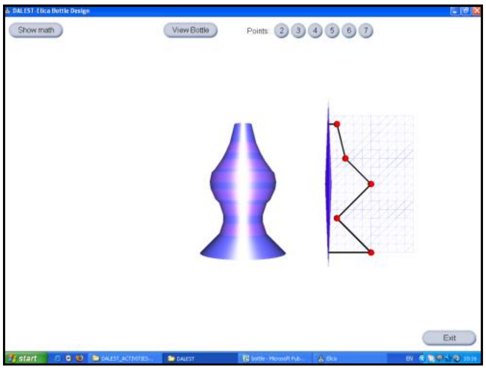
Figure 8. A truncated cone and complementing it to a cone

Their descriptions were of the kind: *a cone that is cut off at the top, a headless cone.* Complementing the cone and considering the solid as a difference of two cones was helpful and made the introduction of the notion of *truncated cone* more natural.

The next problem presented another type of a challenge to the students:

Problem 2. Construct models of the containers like the ones shown below *by Bottle Design:*





For each model, move the upmost point of rotation so that the volume of the new container is larger by 12 cubical unites.

The posing of problems that naturally emerged as a result of the session was interwoven with investigations and discovery of patterns in the context of calculating the volume of rotational solids.

CHOOSING THE *RIGHT* PROGRAM FOR A PROJECT

Being able to identify the most appropriate among several computer applications is an important *soft skill* [5] which our students were given the chance to develop while working on the topic of rotational solids. Here are two examples from [2]:

Problem.

Do you remember the fable of La Fontaine about the Fox and the Stork? (The Fox offered the Stork the soup in a very shallow dish, and the Stork took his revenge by serving the dinner in a very long-necked jar with a narrow mouth.) Choose the most appropriate DALEST application to make computer models of the containers offered by the Stork and the Fox. Is it possible that they could contain the same quantity of food?

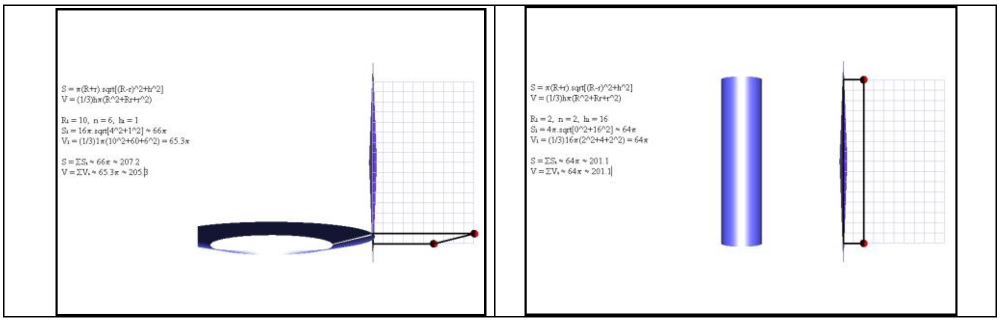


Figure 9. Models of the containers used by the Fox and the Stork by means of two points

Project: Choose an appropriate *DALEST* applications so as to

a) make your own collection of containers, hats, toys
b) model your profile and use it to make a cup of *your own image and likeness*
c) model the *vase of fortune.*

FURTHER IDEAS FOR THE STUDENTS TO EXPLORE

In *Math Wheel* and *Bottle Design* the moving points or the figures under rotation are located in one half-plane according to the rotational axis. The challenge within *Potter Wheel* is the option of crossing this axis (Figure 10.).

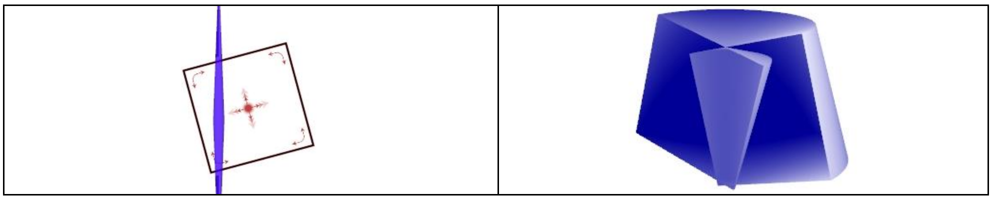


Figure 10. The figure under rotation is intersecting the rotational axis (*Potter Wheel*)

Finding the volume of the rotational solids being obtained; determining the minimal figure generating the same external surface; getting rotational solids with the same external surface but with different intersections are further ideas for explorations. A subject of special interest is the torus. It could be defined as a rotation of a circle (half a circle) and visualized with by means of *DALEST* applications which will be a subject of another didactic experiment (Figure 11.).

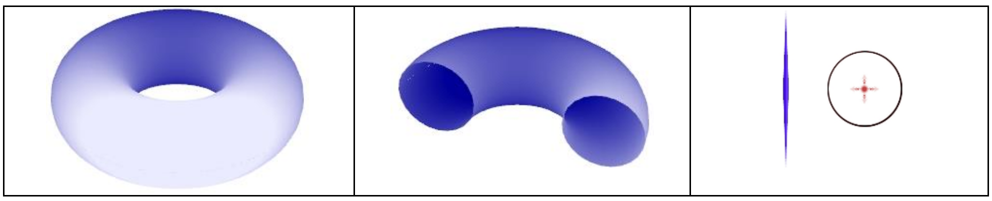


Figure 11. A torus

TEACHERS AS PROJECT AUTHORS

Finally, we shall present some project ideas that emerged in the frames of an in-service teacher training course delivered at the Faculty of Mathematics and Informatics (Sofia University) in the spring of 2008.

The task of the participants was to design a project in which to integrate in a natural way various computer programs (Elica-DALEST, Paint, Power Point, Comenius Logo) so as to illustrate educational ideas of their specific subject.

It was very rewarding for us, the lecturers, to observe the richness of ideas for harnessing the acquired IT skills in activities tuned to the interest of their students.

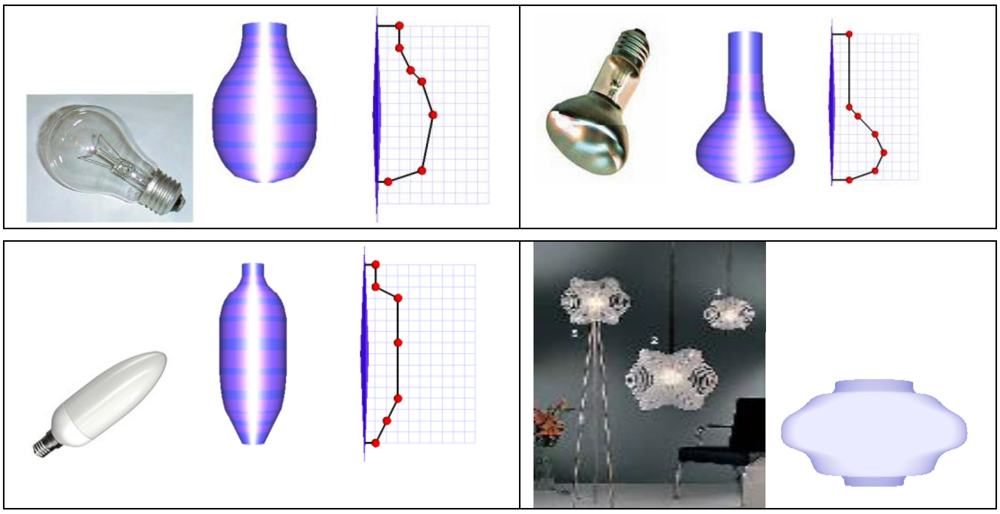


Figure 12. Models of light objects by Marian Radulov

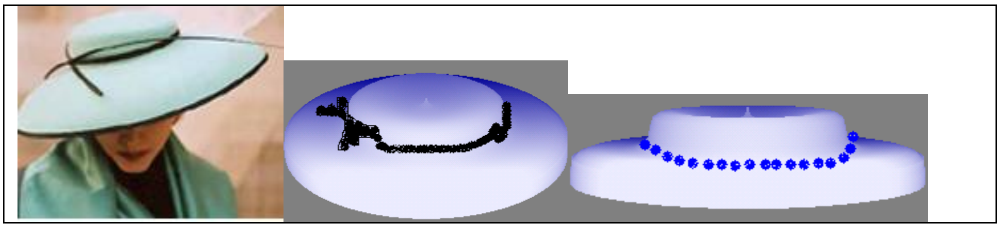


Figure 13. Models of hats by Boyanka Atanasova

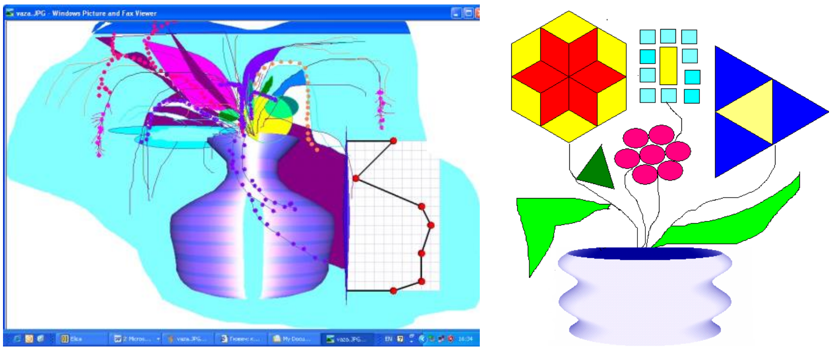


Figure 14. Artistic ideas by Maria Toshkova and Marusia Nikolaeva

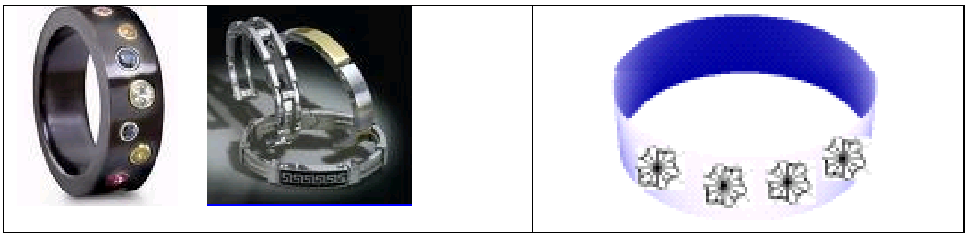


Figure 15. Jewelry design by Diana Tsvetkova

CONCLUDING REMARKS

The experience gained so far makes us optimists since it has shown that when working within the computer application developed for studying rotational solids, the students manage to perform various explorations with more complex solids than the ones present in the traditional curriculum.

As far as in-service teachers are concerned, they could apply their previous expertise and interests in designing and developing projects in the context of rotational solids and IT.

An added value is that work never stops with finding the solution of a problem (or chain of problems) given by the teacher or the lecturer, because the participants are so excited with their achievements that they enthusiastically go on with trying out new ideas of their own.

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Lozanov, C. *Stereometry Activities with Dalest.* University of Cyprus: Nicosia. 2007.
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[4] Boytchev, P., *Elica*, http://www.elica.net
[5] Stefanova E. et al. *When I*Teach means I*Learn: Developing and Implementing an Innovative Methodology for Building ICT-enhanced Skills*, in Benzie D. and Iding M. (eds). *Informatics, Mathematics, and ICT: a 'golden triangle' IMICT 2007* Proceeding, CCIS, Northeastern University, Boston, MA, 2007