MATRIX APPROACH TO SOLVING PROBLEMS IN CONNECTION WITH THE TEACHING CONTENT IN HIGHER SCHOOLS

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ABSTRACT

This paper presents some ideas about how to use the matrix approach in solving problems in regard with the study content of different disciplines taught in the same semester in higher schools as well as the topics within a discipline. To demonstrate the real value of the matrix approach, practical examples are given.

1. INTRODUCTION

In the beginning of our paper [1] we pointed out three factors, which, in our opinion, determine the basic psychological processes in education. These processes are: understanding knowledge, memorizing knowledge and formation of skills for applying it. The first of these factors is: "Presence of understood and memorized relevant knowledge and skills to be used:

1.1. by other disciplines;
1.2. in the hierarchy within each discipline;"

Due to the time limits in presenting the report at that time, we only discussed sub-heading 1.1. Exploring the problem in this sub-heading, we used the "matrix approach" which is not very popular in the field of education. It enabled us to make highly reliable conclusions about the situation with using the inter-disciplinary links. For that purpose, we used the form of a matrix to record the responses to our questionnaire filled out by teachers on a degree programme to the following two questions:

"4. From which major disciplines do you use knowledge when developing your lecture course in … ?
5. In which disciplines, taught at the university, should/could knowledge of your lecture course (discipline) in ……………… be used?"

The answers are systematized in a matrix, part of which is shown in Diagram 1 in [1]. The first row and the first column of this matrix list the disciplines of the degree programme. In each column and row there are 46 boxes. In Diagram 1, only a part of the matrix is presented. The boxes along the diagonal belong to the same discipline and due to the fact that we were interested in different pairs of disciplines, those boxes were left "empty". Every one of all the others was divided by a diagonal, drawn top-down and from left to the right. The cross above the diagonal means that a teacher from the respective column thinks that knowledge from his discipline can be used in the discipline of a teacher, whose number is in the beginning of the respective row. The cross below the diagonal means that the teacher from the respective row presumes that his/her discipline requires knowledge from the discipline of the colleague the column belongs to. What has been said about this matrix means that the information it contains about the study process is not enough. "The emptiness" of the boxes along the diagonal means that in the matrix there is a lack of information about knowledge which has been provided for every topic by its previous topics. Boxes with an asterisk provide information which is far from satisfactory. It is not clear if the topics from which one could use knowledge in his/her topic in a discipline are antecedent. This means, roughly speaking, that for each box from Diagram 1, a proper investigation should be carried out which could enable us to design a new matrix. In the first case, in the first row and in the first column, the topics should be listed in such an order as set in the study schedule. In the second case, in the first row of the matrix the topics from one discipline should be listed against the topics from another discipline – listed in the first column. In other words, in the first case the object of interest should be inter-disciplinary links, and in the second case – the links between to different disciplines. We shall consecutively examine the two cases.

2. MATRIX FOR PRESENTING THE LINKS BETWEEN THE TOPICS IN THE SAME DISCIPLINE

For simplicity of our presentation, we shall take a mathematical discipline spanning over one semester. It is not difficult to consider that the description of that case could be transferred then to a non-mathematical discipline or to other, even two-semester disciplines.

For a one-semester discipline, we put the numbers of the study topics in the first row and in the first column, listed according the sequence they are being taught during the semester. It is clear that, for example, if there are 15 topics, the matrix will consist of 225 boxes. In the columns of every topic, we put square arrows "↥" or "↲" pointing to those rows (columns), from which topics knowledge is used when developing the topic from the respective column (row). This means that, following the didactical rule for “systematics and sequence” there could be square arrows only in boxes along the diagonal of the matrix and above this diagonal. Due to the deductive structure of the Mathematics, divergences in this respect are almost impossible in the matrix for mathematical disciplines. If there are any, we should make corrections in the sequence of the disciplines taught or in their content. This is not like that in many non-mathematical disciplines. We should be conscious of such divergences during the evaluation process (editing) of every handbook. This should be one of the testing criteria during the programme evaluation and accreditation of the respective study fields. Table 1 presents a matrix on the discipline “Lineal algebra and analytical geometry”, which is included in the curriculum of the “Informatics” degree programme in the South-West University “Neofit Rilski”.

Table 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1		⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏		⬏	⬏	⬏	⬏
2			⬏		⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏		⬏
3					⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏		
4					⬏		⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏
5						⬏	⬏	⬏		⬏			⬏		
6			⬏				⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏
7								⬏						⬏	⬏
8														⬏	⬏
9										⬏	⬏	⬏	⬏		
10											⬏	⬏	⬏		
11													⬏		
12													⬏	⬏	⬏
13														⬏	⬏
14															⬏
15															

Thus, the matrix which has been developed, is a precious source of methodical information also for the team teaching the respective discipline. In Table 1 we could see that while working on topic 3, one is using knowledge from the systems linear equations (topic 6), but this knowledge is within the frame of what they have acquired in the secondary school. Often, while re-structuring curricula, it is necessary to merge similar disciplines which have been independent before. In these cases, it is especially useful to develop the already mentioned matrix and, doing so, to be able to discover the links between the particular topics within the merged discipline.

An important factor for understanding every new piece of knowledge is the extent to which all previous knowledge used by the new one has been understood and obtained. Information about this used knowledge gives the square arrows. It's easy to understand that the knowledge we use is divided into basic categories:

- knowledge studied in already passed topics;
- knowledge studied just some minutes ago within the same topic.

In the first case, its successful utilization depends on the following condition – the extent to which it has been acquired before exploring the topic in which they are being used, and also – if they have been properly fixed and sustained during the seminars.

In the second case, there have not been seminar exercises, by which this knowledge to be well acquired. And this is one of the weak points of the strongly established educational system which is a successor of the old Bell-Lancaster's (mutually-teaching) system. This defect of the classical system in higher schools of divided teaching into lectures and seminars is being recently overcome in some universities by eliminating this division into lectures and seminars. In our tradition, this fault to duly fix new concepts, definitions and theorems within the same class session when they have been taught in order to successfully use them in learning the succeeding new knowledge, and this fact was noticed long ago. For example, Prof. Yaroslav Tagamlizki yet in the 50s of the last century used to pose mathematical problems to be solved during lectures. We think that within the currently fixed system of lectures and seminars, this just described defect of using not yet sufficiently acquired knowledge in a given topic when teaching new ones in the same topic can be overcome by posing at least two problems to be solved after every new theorem, which solutions consist of two or three consequent steps in which the newly introduced concepts and theorems are being used. These problems could be solved even only orally. For example:

1. While introducing an equation of straight line between two points, the following problem could be solved: to elaborate the equation of this straight line ***g*** if it goes through the points: a) A(2,3) and B(4,-1); b) C(4,0) and P(0,1). The second case could be used to go to intercept equation.
2. After solving the theorem of Ruche for compatibility of systems of linear equations, there could be solved three systems, so that:
    a. the system is incompatible;
    b. the system is defined;
    c. the system is not defined.

The time used to solve them will be compensated by the much more easy and fast understanding of how to use them when explaining consequent new knowledge in the same topic. In this case, during seminars, problems requiring more complicated solutions will be solved as well as others which train the students in applying mathematical knowledge in non-mathematical situations.

In the matrix of Table 1, this case is being presented by the boxes along the diagonal. The situation, corresponding to every one of these boxes can also be presented in a matrix. In this one, in the first row and in the first column, the new concepts and the new theorems will be listed following the sequence in which they appear in the respective topic. In view of the fact that, in each topic there are usually around 5 to 10 new concepts and theorems being taught, we will put the respective concepts and theorems in the first row and in the first column. Thus, in the matrix there will be around 25 to 100 boxes. And here, if in the boxes, just like in Diagram 1, we use square arrows, presenting exactly which new knowledge is being used in a concrete succeeding new knowledge. There should be arrows only in the boxes above the diagonal. If the arrows appear in the diagonal box, as well, this would be a signal for an existing logical defect, and consequently a psychological circle. The arrows in the boxes above the diagonal demonstrate how to fix the respective new concepts and theorems by short solutions while teaching the topic itself. It is not difficult to find that it is reasonable for such matrices to be used in topics where there are at least 5 new concepts and theorems.

3. MATRIX PRESENTING THE LINKS BETWEEN TWO DIFFERENT DISCIPLINES

Further on, we will consider inter-disciplinary links between the following disciplines – Linear algebra and Analytical geometry, included in the curriculum of a degree programme in "Pedagogy of education in mathematics and informatics". The two disciplines have been taught during the first semester. The Syllabus during the semester has been taken from the Records Book where all the study content is being registered at the Department of Mathematics of the South-West University "Neofit Rilski" (Appendix 2 and 3). The results of the matrix for the links among the topics of the above mentioned disciplines are being presented in Table 2.

In the first row, the numbers of the topics of Analytical geometry are recorded (Appendix 3). It can be seen from the Diagrams that the topics "Rank of a matrix" (LA) and "Equations of straight lines" (AG) are being taught during one and the same week. The same counts also for the topics: "Compatibility of systems linear equations" (LA) and "Equations of straight lines and plane in space" (AG). And we should also note that knowledge which has been acquired by students in the above mentioned disciplines from the course of Linear algebra can be used by the teacher in Analytical geometry while explaining the mutual position of two straight lines and mutual position of two planes. If there is a lack of preliminary co-ordination of the calendar schedules between the two teachers, the teacher of Analytical geometry has two possibilities: either to introduce the theorem of Ruche during the lectures in Analytical geometry or to teach the theorem for the mutual position of two straight lines (two planes) without using it. For example:

Theorem

For the straight lines $\boldsymbol{g_1}$ and $\boldsymbol{g_2}$ given with the equations

(1) $\boldsymbol{g_1 : a_1 x + b_1 y + c_1} = 0$
(2) $\boldsymbol{g_2 : a_2 x + b_2 y + c_2} = 0$

it is given that:

a) $\boldsymbol{g_1 \parallel g_2}$ only when $\boldsymbol{a_1/a_2 = b_1/b_2 \neq c_1/c_2}$;
b) $\boldsymbol{g_1 \equiv g_2}$ only when $\boldsymbol{a_1/a_2 = b_1/b_2 = c_1/c_2}$;
c) $\boldsymbol{g_1} \cap \boldsymbol{g_2} = \{M\}$ only when $\boldsymbol{a_1/a_2 \neq b_1/b_2}$.

Table 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	↵				⬏			⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏
2	↵		↵		⬏			⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏
3	↵		↵		⬏			⬏	⬏	⬏	⬏	⬏	⬏	⬏	⬏
4	↵	↵	↵												
5	↵	↵	↵		↵	↵	↵								
6	↵	↵	↵		↵	↵	↵								
7	↵					↵									⬏
8															⬏
9	↵	↵													
10															
11	↵	↵	↵	↵											↵ ⬏
12	↵	↵	↵	↵											↵ ⬏
13	↵	↵	↵	↵											
14	↵	↵	↵	↵											↵ ⬏
15	↵	↵	↵	↵											↵ ⬏

If the theorem of Ruche has been learned by the students before the topic of the mutual position of two straight lines, a teacher of AG has the possibility to choose whether to prove the above described theorem or to prove the following theorem:

Theorem

For the straight lines g_1 and g_2 given with the equations (1) and (2) it is true that:

A) $g_1 \parallel g_2$ only when $r(A) \neq r(\bar{A})$;
B) $g_1 \equiv g_2$ only when $r(A) = r(\bar{A}) = 1$;
C) $g_1 \cap g_2 = \{M\}$ only when $r(A) = r(\bar{A}) = 2$.

The teacher is also able to demonstrate that the two theorems are equivalent.

Due to the limits of the present work, here we discussed only one of the cases in which the matrix approach is useful in working out the calendar schedule and allocating the study content units of the disciplines considered above.

4. CONCLUSION

In his habilitation work, Prof. Ivan Mirchev in fact got to the idea that for the classical didactical principles for completeness, scientific nature, individual approach, sustainability, systematicalness, succession and visualness it is reasonable to use the new methodological works in methodology research as a criterion for their value and usefulness. As a mathematician, in realizing this idea, he might have used the theorems from mathematics as a solid criterion for truth within a certain number of cases.

It is a trivial truth for every mathematician that after proving the theorem of Pythagoras, it is not necessary to verify it later for every particular rectangular triangle, but directly "step" on it. It is the same as in the methodology – after we demonstrate that a certain methodological approach assures the implementation of some of the didactical principles, it is not necessary to verify its usefulness and reliability by way of an experiment. An experiment is necessary for cases for which there is not enough scientific knowledge. For example, for a person who knows the characteristics of a regular glass very well, it is not necessary to verify experimentally if a product of such glass will break if thrown against a stone from 2 meters of height.

In our case, if we accept the reliability of the didactical principles, it is enough only to demonstrate that the idea discussed above is in line with some of them. Indeed, it guarantees the implementation of the principles for visibility, systematicalness and succession and from here we get to accessibility and sustainability of knowledge. This means that for people who know the specific characteristics of learning process in mathematics very well, it is not necessary to carry out whatsoever experiments in order to "prove" the usefulness and reasonability of the present work.

Appendix 1

LINEAR ALGEBRA AND ANALYTICAL GEOMETRY
Syllabus
Academic Year 2008-2009

Week	*Topic*
1	Elements of combinatorics
2	Matrices. Types. Operations with matrices
3	Determinants. Properties
4	Vectors and operations with them
5	Rank of a matrix. Elementary transformations of a matrix
6	Systems of linear equations
7	Equations of straight lines and planes
8	Linear spaces
9	Linear transformations (operators)
10	Eigenvectors and eigenvalues of linear transformations. Characteristic roots of a matrix
11	Orthogonal matrices and orthogonal transformations
12	Symmetric matrices and symmetric transformations
13	Quadratic forms. Canonization
14	Second degree curves
15	Second degree surfaces

Appendix 2

LINEAR ALGEBRA
Syllabus
Academic Year 2008-2009

Week	*Topic*
1	Matrices. Types. Operations of matrices
2	Determinants from second, third and n-th order
3	Minors
4	Inverse matrix. Properties
5	Rank of a matrix
6	Systems of linear equations. Consistent and inconsistent systems of linear equations
7	Cramer’s Formulas. Gaussian Elimination Method. Homogeneous systems of liner equations

8	Linear spaces
9	Isomorphism between linear spaces
10	Linear transformations (operators)
11	Eigenvectors and eigenvalues of linear transformations. Characteristics roots of a matrix
12	Euclidean and Unitarian spaces. Isomorphism
13	Orthogonal matrices and orthogonal transformations
14	Symmetric matrices and symmetric transformations
15	Quadratic forms. Canonization

Appendix 3

ANALYTICAL GEOMETRY
Syllabus
Academic Year 2008-2009

Week	***Topic***
1	Directed segment. Free vector
2	Addition and subtraction of vectors. Multiplication of a vector by a scalar. Properties
3	Linear dependency and linear independency of vectors
4	Vectors and triple scalar product of vectors
5	Equations of a straight line
6	Equations of straight line and plane in space
7	Equation of a circle. Conic sections
8	Cylindrical, conic and rotation surfaces
9	Homogeneous co-ordinates. Complex points, straight lines and planes. Metrics concepts
10	Double relation of four points and four planes. Harmonica groups
11	Second degree surfaces
12	Polarity in relation with second degree surfaces. Projective canonical equations of second degree surfaces
13	Affine properties of the second degree surfaces
14	Metrical properties of the second degree surfaces
15	Metrical canonical equations of the second degree surfaces

REFERENCES

[1] Ganchev I., Gyudzhenov I., "Matrix Approach in Solving Problems of Interdisciplinary Relations in Higher Schools", Proceedings of the Thirty Seventh Spring Conference of the Union of Bulgarian Mathematicians "Mathematics and Education in Mathematics", Borovets, April 2-6, , pp. 95-103, (2008).

РЕЗЮМЕ

В работата са разгледани идеи за използването на матричния подход за решаване на проблеми, свързани с учебното съдържание на дисциплини във висшите училища, които се изучават в един и същи семестър. Авторите дават практически примери и демонстрират ценността на матричния подход.