THE IMPACT OF ICONIC REPRESENTATIONS IN SOLVING MATHEMATICAL PROBLEMS OF THE FRACTIONS DIVISION

Charalambos Efstathiou*, Androulla Petridou**
Athanasios Gagatsis* & Panagiota Kalogirou******
Department of Education, University of Cyprus
e_pampos@hotmail.com *, andri_petridou@hotmail.com **
gagatsis@ucy.ac.cy ***, pamelakalogirou@yahoo.gr ****

ABSTRACT

The present study investigates the role of the four iconic representations (decorative, auxiliary-representational, auxiliary-organizational and informational) in the problem solution procedures of fraction division problems, in measuring and sharing, from 6^{th} grade students. For the purpose of the study, 100 pupils were asked to complete a questionnaire with verbal problems accompanied by iconic representations. The results indicate that the decorative, organizational and informational pictures have negative influence in students' behavior among both measurement and sharing tasks. Generally, iconic representations affect negatively the solution procedures of fraction division problems, except the representational picture in measurement.

THEORETICAL AND EMPIRICAL BACKGROUND

The research in the space of mathematical education in the past few years is turned intensely in the theory of representations. According to Duval (2006) a mathematic significance cannot become comprehensible via the senses, but only through a representation (report in Deligianni, Ilia and Panaoura, 2007). The reason for which particular accent in the significance of representations is given, in the space of the instructive mathematical education, is the fact that the representations are considered inherent with the mathematics (Dufur-Janvier et al, 1987, p.110; Kaput 1987 p.25). In certain cases, the representations are so much closely tied up with a mathematic significance, that it is difficult the significance to become comprehensible with the absence of particular representations. Special importance is given in the representations when they are used in the teaching of difficult subjects, as is the region of fractions and still more in the division of fractions. A lot of researchers (Brousseau et al., 2004; Kieren, 1993; Lamon, 1999; Charalambous and Pitta-Pantazi, 2007), report that the fractions are considered as the most complicated numbers and the division the most complex of the mathematical equations (report in Kaboyzi, Michael, Xenofondos and Gagatsis, 2008).

According to a lot of researchers, representations are distinguished in exterior and internal. The internal representations include likely intellectual shapes, which build the individuals in order to represent the reality. It is not immediately notice and exists with through the exterior behaviour of the persons (Markou and Gagatsis, 2002). The term "exterior representations" is reported in the all exterior symbolic institutions, which aim at they represent externally a concrete mathematic reality. According to Lesh, Post and Behr (1987) there are five different types of systems of exterior representations concerning the learning of mathematics and the resolution of problem. These types are the text, the manipulative, the object-model, picture or diagram, language and symbol. Fundamental objective of teaching should constitute the faculty of translation from a system in the other. The term 'translation' is reported in the psychological processes that are involved in the passage from one representation to other (Janvier, 1987).

The virtual representations, recommend a tool that plays important role in the mathematical education as means support of thought, communication and transmission of mathematic significances. Carney and Levin (2002) proposed a categorisation of pictures depending on their operation. Base of this categorisation, Theodoulou, Gagatsis and Theodosiou (2003) proposed a similar categorisation depending on the operation of pictures in the resolution of mathematic problem. In the present research is followed the discrimination of virtual representations according to this classification, which becomes depending on the operation that possess the pictures in the treatment of texts. Thus the virtual representations are distinguished in decorative, auxiliary-representational, auxiliary-organisational and informative.

The decorative pictures do not give any information on the resolution of the problem. In the auxiliary-representational pictures is included entire or part of content of problem. However, those pictures are not essential for the resolution. Auxiliary-organizational pictures help the students to solve the problem by guiding them to organize the given statements of the problem. Finally, informational pictures provide information that is essential for the solution of the problem; in other words, the problem is based on the picture.

The fractions, in general, as significance, are very abstract. According to Philipou and Christou (1995), the fraction can be interpreted with three ways: as part of all, as division and finally as reason. Southwell (1985), reports that the fractions are symbolized with completely different ways from the entire numbers and their algorithms require completely new processes. (reported in Gagatsis, Michailidoy and Siakalli, 2001).

The division of fractions is considered as one of the most difficult subject of mathematics in elementary school (Nikolaou, Loizou, Stylianu, Gagatsis, 2006). The records of students are very low, due to the fact that the students often considered the action of division in the fractions as most mechanic and less comprehensible subject of teaching in the elementary school (Ma, 1999; Bulgar, 2003; Tirosh, 2000; report in Kabouzi, Michael, Xenofondos and Gagatsis, 2008).

Moreover, Vergnaud (1996) reports that the representations are used in the elementary school, for the approach of fractions significances and the construction of explicit numbers, play decisive role in the importance of significance and constituting component of mathematic significances (report in Kabouzi, Michael, Xenofondos and Gagatsis, 2008). In order to render the abstract significances more accessible, the teachers use variety of exterior representations, as pictures, diagrams and manipulative objects. But does the symbolic means of representation contribute positively or not in the comprehension of mathematic significances? (DeLoache et al., 1998; Dufour - Janvier et al., 1987; Seeger, 1998; Kleanthous, Pachiti and Ilia, 2003; report in Kabouzi, Michael, Xenofondos and Gagatsis, 2008).

Konstantinou, Parisinos, Stylianou and Gagatsis (2007), conclude that the children do not understand as well all problems with the same type of representation and that does not exist “ideal” way in order to represented a problem so as to they perceive better. Also, they propose that the teacher will be supposed daily to use variety of representations of problems, so that it achieves the better possible comprehension from the students.

The teachers play a very important role in the choice of suitable representations that they use in their teaching. This can be achieved by asking the students what they think and then asking them to represent their thoughts with various ways. The students should comprehend that the representations of teacher are not unique and the more equitable way of representation of significance. The objective is the children to have the occasion to think and to select through an abundance of representations in the search of meaning of mathematics that is taught (Fennell & Rowan, 2001).

THE STUDY

PURPOSE

The purpose of this study was to investigate the role of four different modes of representation (decorative [DP], auxiliary-representational [RP], auxiliary-organizational [OP] and informational [IP]). More specifically the aim of the study was to explore and compare the effects of those representations picture in the solution procedures of fraction division problems in measurement and sharing.

METHODOLOGY

PARTICIPANTS

The sample of the research consisted of 100 students of 6th grade (10-11 years old) from four elementary schools in three districts of Cyprus. The sample was selected with convenience sampling method. The students came familiar with the subject only the last semester of 5th grade.

DATA COLLECTION

In order to collect the data needed for this study, a questionnaire was constructed. The questionnaire consisted of 10 problems with fraction division. All the problems were solving with division of integer with fraction. The problems 1,2,4,7 and 9 were measurement problems and the problems 3,5,6,8 and 10 were sharing problems. The problems 2 and 5 were verbal, 3 and 4 were accompanied with DP, 1 and 8 with RP, 6 and 9 with OP and 7 and 10 with IP. Students had to decide which one of the four given choices (right answer, multiplication, addition and wrong division) was the correct one. Furthermore, students had to declare in which degree the picture helped them.

PROCEDURE

The written questionnaire was administered to the students in usual classroom conditions. Students were asked to solve all the items by circle their answer. They were not obliged to use the pictures that accompanied the problems. Actually, they were instructed to use the representations if they believed that they could help them resolve the problems. Students were given 40 minutes to solve the problems.

VARIABLES OF THE STUDY

The variables of the study were the following:

M:	Sharing problem	v:	Verbal problem
C:	Measurement problem	d:	With decorative picture
S:	Wrong strategies	r:	With auxiliary-representational picture
	m: Multiplication	o:	With auxiliary-organizational picture
	a: Addition	i:	With informational picture
	w: Wrong division	B:	Students’ Beliefs

SCORING OF THE TASKS

Problem tasks were scored as follows: 0=wrong answer or no answer and 1=correct answer. The students' beliefs for each picture were scored as follows: 1=most, 0,75 = lots, 0,25=little and 0=not at all.

METHOD OF ANALYSIS

For the analysis of the collected data, Gras's Implicative Analysis by using the computer software C.H.I.C. was performed. Also, it was used Excel's worksheets.

RESULTS

A basic aim of the study was to examine whether different modes of representation (decorative, auxiliary-representational, auxiliary-organizational and informational) affect students' performance in solving fraction division problems. Figure 1, shows the students' performance in each problem tasks of the questionnaire. The results show that the percentages are higher in all the measurement problems tasks in compare with the respectively sharing problems. The highest percentage (73,47%) is observed at the measurement's problem with RP, whilst the lowest percentage (18,09%) refers to the sharing problem with OP. This may due to the structure of the OP, which may confuse students instead of help them to organize their thought. Also, according to the graph, the problems with DP confused student, both in measurement and sharing. Students score higher at verbal problems. Although the structure of the DP is not useful at all for the solution of the problem, doesn't mean that is not affect student's reaction in problem solving. Finally, in the problems with IP students scored low but not the lowest of the entire questionnaire.

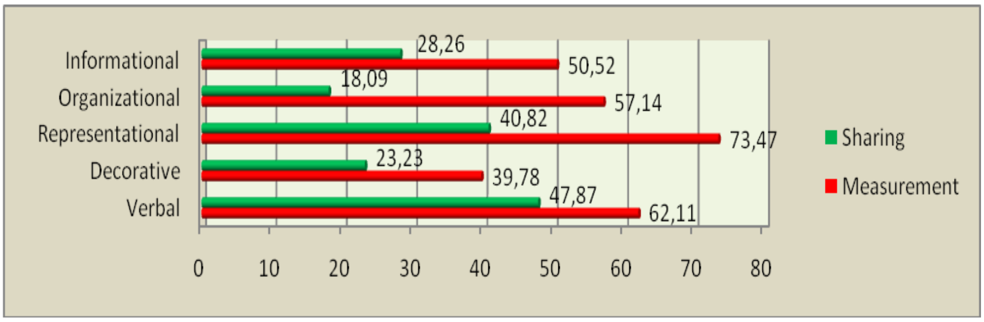


Figure 1: *Percentages of success in each different representational mode.*

SIMILARITY BETWEEN THE TASKS

The Similarity Diagram (Figure 2) shows how tasks are grouped according to the similarity of the ways in which they have been solved. Also, shows how tasks are grouped to the similarity of the students' beliefs towards pictures. The similarities in bold color are important at level of significance 99%.

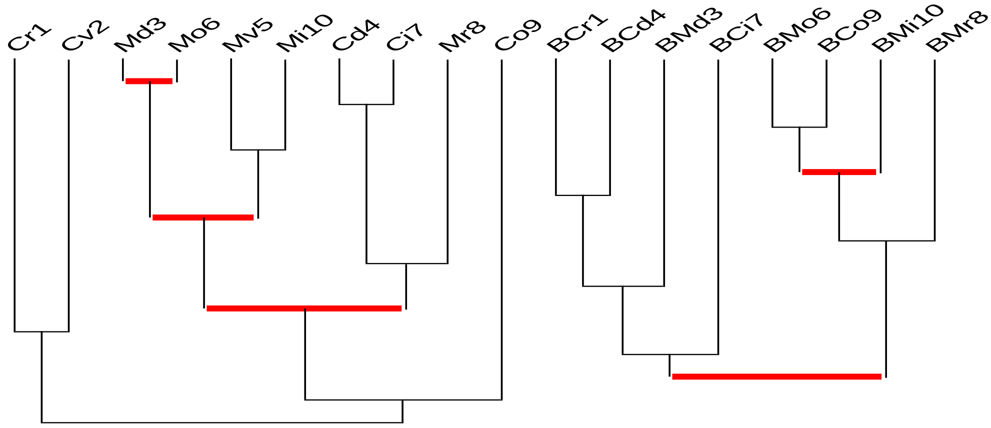


Figure 2: *Similarity diagram of students' responses to the tasks*

According to the Similarity Diagram (Figure 2), two groups are clearly distinguished. The first group consists of the variables Cr1, Cv2, Md3, Mo6, Mv5, Mi10, Cd4, Ci7, Mr8 and Co9, which represent students' answers efficiency in solving the problem tasks. The variables BCr1, BCd4, BMd3, BCi7, BMo6, BCo9, BMi10 and BMr8 are included in the second similarity group that concerns students' beliefs towards pictures.

In the first group, two subgroups are distinguished. The first subgroup consists of the variables Cr1 and Cv2 which represent the students' efficiency in measurement, verbal problem and the measurement problem with RP. In the second subgroup, two other subgroups are distinguished. The first group, of this subgroup consists of the variables Md3, Mo6, Mv5, Mi10, Cd4, Ci7, Mr8 and Co9. In this particular subgroup of similarity, the stronger relation lie among the task Md3 and Mo6 (0,998). This relation refers to the sharing problem with DP and the sharing problem with OP. In those problems students' behaviors is the same. Another important relation lie among the task Cd4 and Ci7 (0,997), which refers to measurement problems with DP and IP. All the sharing problems are grouped, except Mr8, which refers the problem with RP. Students may handle this kind of problems with a similar way.

In the second group, two subgroups are also distinguished. The first subgroup, consists all the measurement problems (BCr1, BCd4, BMd3 and BCi7) except the problem with the OP. Also, include the sharing problem with DP. Strong relation occurs among BCr1 and BCd4 which refers to the measurement problems with RP and DP respectively. The second subgroup consists all the sharing problems (except the problem with DP) and the measurement problem with OP. In this group there is a similarity relation between BMo6 and BCo9 (0.990), which refers the OP in sharing and measurement. This fact, states that students have the same beliefs about the OP, whatever the problem is (sharing or measurement).

IMPLICATIVE GRAPH

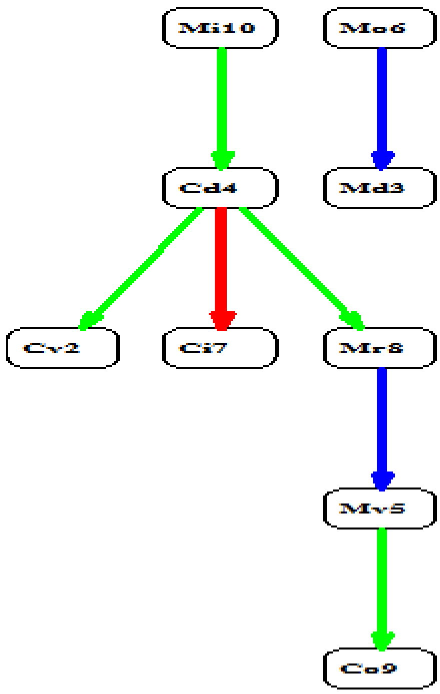


Figure 3: Implicative graph of students' responses to the tasks

The Implicative Graph (Figure 3) shows the implications between problem tasks with picture or not. According to the Implicative Graph (Figure 3), two groups are clearly distinguished. In the first group, appears an in-relational relation between the variables Mo6 and Md3. All the students who can solve the sharing problem with OP can also solve the sharing problem with DP. In the second group, the most difficult problem was Mi10 which refers the sharing problem with IP, whilst the easiest was the Co9, which refers to the measurement problem with OP. According to the Implicative Graph (Mi10, Cd4, Mr8, Mv5, Co9) the success at the sharing problem with IP implies the success to the other four problems. There is an in-representational relation between the two problems with IP. The student who can solve the sharing problem with IP they can also solve the measurement problem with the same kind of picture. Maybe, the IP helps the students whatever the kind of the problem is. The implication of Ci7 from Cd4 is the most powerful implication (statistic significance 99%). There is an in-relational relation between the measurement problems with IP and DP, whilst the last one appears to be the most difficult. The success in problem Cd4 implicate the success toward the others measurement problems, except the problem with RP which does not connected with implicational relations, because it has very high percentages of success.

DISCUSSION

The purpose of this study was to investigate the role of four different modes of representation (decorative, auxiliary-representational, auxiliary-organizational and informational). More specifically, the aim of the study was to explore and compare the effects of those representation pictures in the solution procedures of fraction division problems in measurement and sharing. As Vergnaud (1996) reports that the representations are used in the elementary school, for the approach of fractions significances and the construction of explicit numbers, play a decisive role in the importance of significance and constituting component of mathematic significances (report in Kabouzi, Michael, Xenofondos and Gagatsis, 2008).

According to the results of our research, an important difference occurs in students' performance in measurement and sharing problems in all kinds of pictures. Generally, students score higher in measurement problems than in sharing problems with the same picture. This is due to the absence of sharing problems from school textbooks. Also, the percentages in all the problems are generally not so high, due to the difficulty of the subject of fraction division, even in measurement.

Iconic representations appear to affect students' performance. DP, OP and IP seem to have a negative influence in both measurement and sharing. The representation which helps most is RP, both in measurement and sharing. But, in measurement problems this picture increases the performance of the students in relation with the verbal problems. On the other hand, RP decrease the performance in sharing problem, in relation with the verbal problem, but to a lower degree in relation to the reductions of the other sharing problems. Generally, all kinds of pictures seem to decrease the performance in sharing problems. Furthermore, although DP are only a decorative part for a problem, without giving any information for the solutions, however they impede it. OP in sharing problems affect very negatively the performance of the students. In conclusion, iconic representations affect the finding of solutions in problem-solving in fraction division negatively, except the RP in measurement problems. The beliefs of the students differ for each kind of iconic representation. Students believe that the DP both in measurement and sharing are not helpful to find out the solution. This was expected according to Theodoulou, Gagatsis and Theodosiou (2003), this kind of picture does not give any information for the solution. Moreover, students believe that IP in both measurement and sharing are very helpful. Theodoulou, Gagatsis and Theodosiou (2003), support that the structure of IP gives all the necessary information for the solutions of a problem. Specifically, students' belief does not correlate with their performance in any kind of pictures. This may be due to the difficulty of the fraction division.

This study researched the effect of the iconic representations in measurement and sharing problems in integer division with fraction. Further research is needed to investigate the affect of the representations in fraction's division with integer, fraction with fraction, with simple and abused fraction or with varied numbers.

Furthermore, the same research may take place with larger amount of students and different ages. Finally, this research can also be done right after the teaching of fraction division.

REFERENCES

Carney, R. N., & Levin, J. R. (2002). Pictorial illustrations still improve students' learning from text. *Educational Psychology Review, 14*(1), 5-24.

Deligianni, E., IIia, I., & Panaoura, A. (2007). Faculty of use of multiple representations in the fractions. In A. Gagatsi, *Problems of learning of Mathematics at the passage from the Public school in the High school.* Nicosia: University Cyprus, Faculty of Social Sciences and Sciences of Education (In Greek).

Dufur-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical ConsiderationsConcerning the Problem of Representation. In C. Janvier (Ed.), *Problem of Reprsentation in the teaching and learning of Mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

Fennel, F., & Rowan, T. (2001). Representations: An Important Process for Teaching and Learning Mathematics. *Teaching Children Mathematics*, 7 (5), 288-292.

Gagatsis, A., Michailidou, E., & Siakalli, M. (2001). *Theories of Representation and Learning of Mathematics*. Nicosia: ERASMUS IP 1.

Janvier, C. (1987). Translation processes in mathematics education. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 27-32). Hillsdale, NJ: Lawrence Erlbaum.

Kabouzi, I., Michael, A., Xenofondas, P., & Gagatsis, A. (2008). The use of representations in the resolution of problems of division of fractions. In A. Gagatsis, P. Damianou, A. Philipou, *10th Cyprus Congress of Mathematic Education and Science.* (pp.299-313). Pafos, Cyprus: Cypriot Mathematics Company (In Greek).

Kaput, J.J. (1987). Representation System in Mathematics. In C. Janvier (Ed.), *Problems of representation in the Teaching and Learning of Mathematics* (pp.19-26). Hillsdale, NJ: Lawrence Erlbaum. Larkin.

Konstantinou, K., Parisinos, M., Stylianou, M., & Gagatsis, A. (2007). Types of representations and resolution of problems with fractions - Convictions of Students. In A. Gagatsis, I. Ilia, X. Lemonidis, *9th Cyprus Congress of Mathematic Education and Science*. (pp.207-220). Pafos, Cyprus: Cypriot Mathematic Company (In Greek).

Lesh, R., Post, T., & Behr, M. (1987) Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning mathematics* (pp. 33-40). Hillsdale, N.J.: Lawrence Erlbawm Associates.

Markou, A., & Gagatsis, A. (2002). Representations and Learning of Mathematic Significances: Application in the fractions. In A. Gagatsi & G. Makridi: *E' Cyprus of Congress of Mathematic Education and B' Conference of Astronautics and Interval*. Cypriot Mathematic Company (In Greek).

Nikolaou, A., Loizoy, A., Stylianou, M., & Gagatsis, A. (2006). Virtual representations and resolution of problems of division of fractions. In A. Gagatsis, A. Panaoura, P. Damianou, *8th Cyprus Congress of Mathematic Education and Science.* (pp.175-186). Pafos, Cyprus: Cypriot Mathematics Company (In Greek).

Theodoulou, R., Gagatsis, A. & Theodosiou, A. (2003). Are pictures always helpful in mathematics problem solving? In A. Gagatsis & I. Ilia (Eds), Representations and Geometrical models in the Learning of Mathematics, Vol. 1 (pp. 67-85). Nicosia: Intercollege Press (In Greek).

Σκοπός της παρούσας έρευνας είναι να ερευνήσει την επίδραση των τεσσάρων ειδών εικονικών αναπαραστάσεων (διακοσμητικών, βοηθητικών - αναπαραστατικών, οργανωτικών και πληροφοριακών) ως προς τον τρόπο επίλυσης προβλημάτων διαίρεσης κλασμάτων (μέτρησης ή μερισμού), από μαθητές Στ' τάξης. Εκατό μαθητές κλήθηκαν να συμπληρώσουν ένα δοκίμιο, το οποίο αποτελείτο από προβλήματα, με ή χωρίς εικονική αναπαράσταση (μερισμού και μέτρησης). Τα αποτελέσματα της έρευνας, έδειξαν ότι οι διακοσμητικές, οι οργανωτικές και οι πληροφοριακές εικόνες επηρεάζουν αρνητικά τη συμπεριφορά των μαθητών και στα δύο έργα. Γενικά, οι εικονικές αναπαραστάσεις επηρεάζουν αρνητικά την εύρεση τρόπου επίλυσης των προβλημάτων διαίρεσης κλασμάτων, εξαιρουμένης της βοηθητικής-αναπαραστατικής στη μέτρηση.

APPENDIX

Parts from the test

3. It takes 3 hours for a bus to do $\frac{1}{2}$ of its route. How long does the bus take to do the entire route?

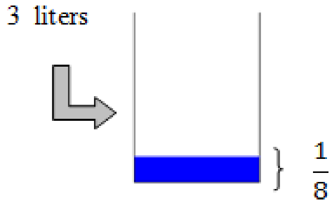
A	B	Γ	Δ
$\frac{1}{2}+\frac{1}{2}=v$	$\frac{1}{2}:3=v$	$3\times\frac{1}{2}=v$	$3:\frac{1}{2}=v$

Not at all	Little	Lots	Most
1	2	3	4

5. The $\frac{1}{8}$ of a number is 5. Which is that number?

A	B	Γ	Δ
$5:\frac{1}{8}=v$	$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=v$	$5\times\frac{1}{8}=v$	$\frac{1}{8}:5=v$

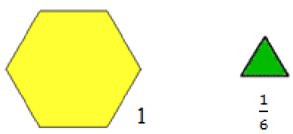
6. 3 liters of water was put in a pot. Now, only the $\frac{1}{8}$ of the pot is full. How many liters can seat in all the pot?



A	B	Γ	Δ
$3\times\frac{1}{8}=v$	$\frac{1}{8}:3=v$	$3:\frac{1}{8}=v$	$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=v$

Not at all	Little	Lots	Most
1	2	3	4

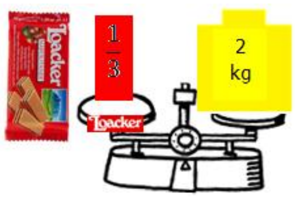
9. How many times the $\frac{1}{6}$ go into 1?



A	B	Γ	Δ
$1:\frac{1}{6}=v$	$\frac{1}{6}:1=v$	$\frac{1}{6}+\frac{1}{6}=v$	$1\times\frac{1}{6}=v$

Not at all	Little	Lots	Most
1	2	3	4

10. How much the whole chocolate weigh?



A	B	Γ	Δ
$2\times\frac{1}{3}=v$	$\frac{1}{3}+\frac{1}{3}=v$	$2:\frac{1}{3}=v$	$\frac{1}{3}:2=v$

Not at all	Little	Lots	Most
1	2	3	4