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# PRIMARY MATHEMATICAL MODEL OF THE HEAT EXCHANGE FOR THE RF-EXCITED GAS DISCHARGE IN HE-CD LASER SYSTEM. Part II – NUMERICAL RESULTS

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This paper presents numerical simulations and results, obtained by means of the mathematical model proposed in the part I. The profile of the gas discharge temperature in a cross-section of the laser tube under various operating conditions are carried out. Engineering solutions to improve the heat-exchange and to increase the rate of applied power up to 35-40% are proposed.[1]

## Numerical Simulations and Discussions.

The mathematical model (I.4)-(I.8) was applied to describe the heat-exchange of the gas discharge and its surroundings [1]. The calculations were carried out many times for different selection of the parameters in order to simulate various boundary conditions and to improve the laser system efficiency.

Some of the obtained results for the gas temperature distribution $T_g$ on the line a-a are shown in Fig.1. In the case of natural convection (v=0) it can be seen that the maximum value $T_{max}$ of the temperature in the gas discharge was slightly displaced to the direction of the "higher" electrode. This effect can be accounted for by a disbalanced distribution of the charged particles and nonsymmetrical intensity of the electric field in the cross-section of the tube at nonsymmetrical values of the voltage (see also [2], [3], [4] and [5]).

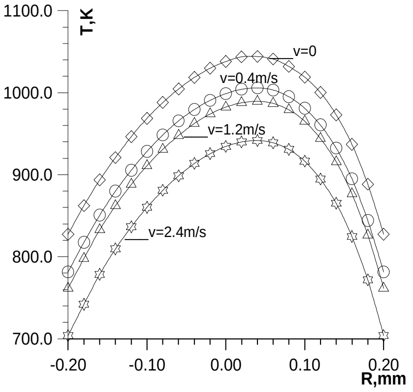

Fig.1 Distribution of gas temperature $T_g$ on the line a-a for natural convection (v=0) and for forced air convection at v=0.4, 1.2, and 2.4m/s.

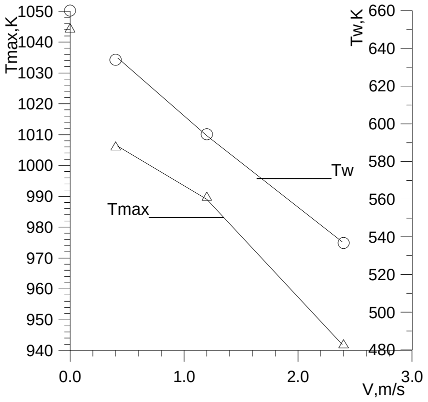

Fig.2 Variations of the surface gas temperature $T_w$ and highest temperature $T_{max}$ in the cross-section of laser tube at forced cooling and $P$=const. On the left ordinate are shown $T_w$ and $T_{max}$ at (v=0).

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[1] AMS Subject Classification. 81V80, 78A60

The increase of the applied electric power depends on the active heat exchange. For this reason the laser tube can be cooled for instance by airflow. Hence, the gas temperature $T_g$ in the cross-section and also the surface gas temperature $T_w$ will be decreased. The ratio of the heat transfer by convection and the heat transfer by radiation from the outside surface of the laser tube will be changed. The calculations were carried out for three values of the airflow: v=0.4, 1.2 and 2.4m/s. The heat transfer coefficient $\alpha$ was found from (I.15). Then, by using the boundary condition (I.2) the rates for $T_w$ were calculated (see Fig.2). The ratio $\eta$ increases while the surface gas temperature $T_w$ decreases. At the airflow velocity v=1.2m/s we have $\eta=0.613$, so this process becomes predominant. Because of all mentioned above the common reduction of the gas temperature $T_g$ in the cross-section of the laser tube takes place, including its highest value $T_{max}$ (see Fig.1). For juxtaposition on the left ordinate axis in Fig.2 the values of $T_w$ and $T_{max}$ at a natural convection are also given.

The increase of the exchanged heat is of use only if the applied electric power increases at an optimal permanent temperature. Otherwise, the temperature decrease can involve a deterioration of the qualitative discharge structure or laser generation collapse. The applied power at a forced cooling can be augmented while the corresponding highest gas temperature value $T_{max}$ attains to the maximum gas temperature $T_{max}$ for natural convection, which is considered to be optimal.

In order to determine a new value of the applied power P we carried out the next numerical procedure: many times we solve the model (I.4),(I.8) by successive increase of P and compare the highest obtained value of the gas temperature $T_{max1}$ with the initial optimal maximum $T_{max}$ at v=0 while $T_{max1}< T_{max}$.

The values for the ratio $\eta$ show that nevertheless of the increase of P, a redistribution of the transferred heat to the atmosphere is in advantage for the heat exchange by convection. In Fig.3 the variation of the surface temperature $T_{w3}$ is given. The continuous increase of P while the optimal gas temperature $T_{max}$ is attained by $T_{max1}$ leads up to the equalization of the surface temperature at air-flow cooling and the initial surface temperature without cooling. It was established that because of the active heat exchange and the consequential applied power increase the thermal profile in the cross-section of the laser tube was changed. This effect can be explained by more rapid increase of the heat transfer coefficient $\alpha$ with respect to the augmentation of airflow cooling velocity v. On the other hand if the surface temperature $T_{w3}$ becomes less than the Cd-reservoir temperature, the metal vapor condensation on the walls is possible. This will inevitably involve the reduction and deterioration of laser generation. So, natural restrictions on the forced convection rates must be imposed. In Fig.3 we can also see the increase of the ratio $P/P_0$ in measureless units, with P being the applied electric power at air-flow cooling and $P_0$ being the initial applied power without cooling (i.e. v=0). For a fixed optimal $T_{max}$ the increase of $P/P_0$ is possible up to 25%.

## Heat Exchange Intensification by Rectangular Section Fins.

One of the ways to increase the exchanged power to the surroundings is to extend the outside surface area F of the laser tube [6]. For this reason we will assume the laser electrodes to be extended on the outside in the form of copper rectangular cross-section fins. The fins on the surface give a larger outside surface area for the same internal surface area, and hence increase the cooling effect for a given volume. Consider the element of the electrode in Fig.4. The main problem is to find the fin efficiency of heat exchange $\alpha_p$ [7]:

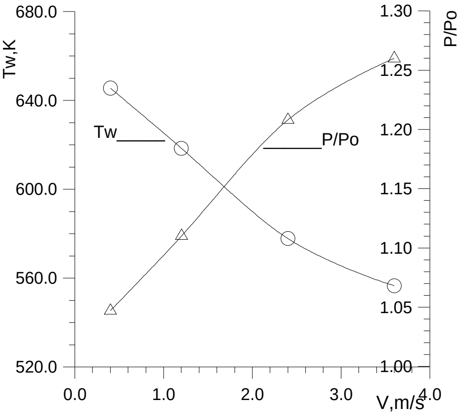

Fig.3 Variations of the surface gas temperature $T_w$ and ratio $P/P_0$ at forced cooling and a constant maximum gas temperature $T_{max}$.

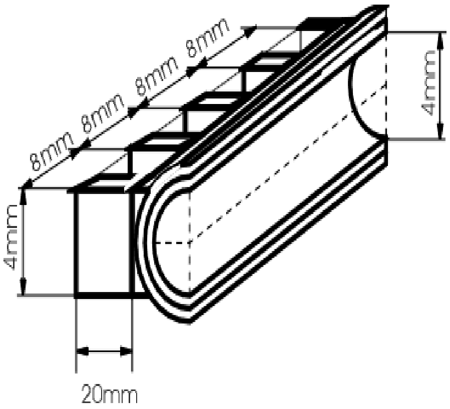

Fig.4. Extended surface area of the electrodes by rectangular cross-section fins.

$$\text{(II.1)} \quad \alpha_p = (\alpha_R.F_R + \alpha_w.F_w)/F \;,$$

where $\alpha_R$ is the heat transfer coefficient of the fins,.$\alpha_w$ - the coefficient of the outside laser tube without fins, $F_R$,$F_w$ - the surface areas of fins or without fins, $F = F_R + F_w$.

At a natural convection for vertical plate fin of high l=4mm, the Nusselt number holds[8]: $Nu = 0.695(Gr.Pr)^{0.25}$,

where Gr is the Grasshof number (I.11) and Pr=0.733 is the Prandtl number for airflow [6]. For the same values of $T_w$, $\beta$, $\lambda$, $\Delta T$ and $\nu$ as in the previous section by using (I.9)-(I.11), we obtain Gr=3123.67, Nu=5.2 and $\alpha_R$=32.6W/(m$^2$.K), with common surface area $F_R$=0.008 m$^2$. For $\alpha_w$=17.61W/(m$^2$.K) and $F_w$=0.023m$^2$ from (II.1) we find $\alpha_{ð}$=21.55W/(m$^2$.K).

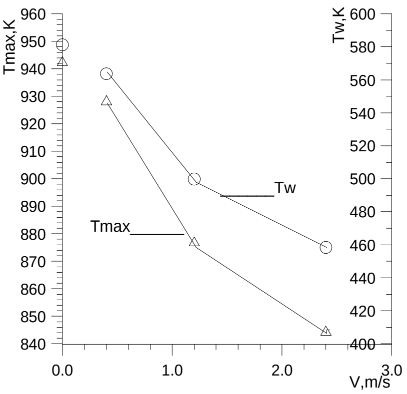

Fig. 5. Surface gas temperature $T_w$ and highest temperature $T_{max}$ in the cross-section of laser tube at forced cooling and applied power $P$=const for the case of metal rectangular fins. On the left ordinate are shown the values of $T_w$ and $T_{max}$ at natural convection (v=0).

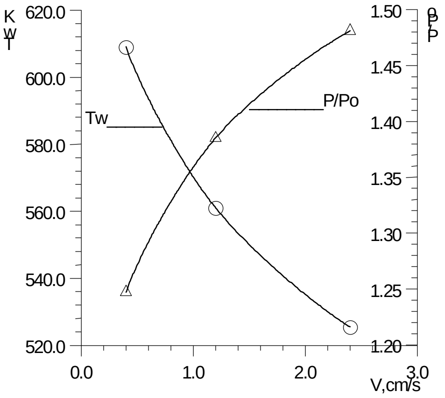

Fig. 6. Variations of the surface gas temperature $T_w$ and the ratio of applied powers $P/P_0$ at forced cooling and a constant maximum gas temperature $T_{max}$ for the extended laser surface area by metal rectangular fins.

The increase of the outside surface area involves linear density change of the heat flow $q_l$. A new reduced linear density of the heat flow is $q_l^{'} = \frac{Q}{l} . \frac{F_w}{F_w + F_R}$. Then we continue by solving the boundary-value problem (I.4),(I.8) in this case. The values of $T_w$ and $T_{max}$ are shown in Fig.5. on the left ordinate. For the case of forced air-cooling of vertical metal fins if $Re<10^5$, the Nusselt number is [8] $Nu=0.644\ Re^{0.5}Pr^{0.33}$. By using this formulae, (I.13) and (I.9) we find $Re=254.77v$, $Nu=9.18v^{0.5}$ and $\alpha_R=57.6v^{0.5}W/(m^2.K)$. Now from (I.15) and (I.1):

(II.2) $\alpha_p = 14.86\ v^{0.5} + 24.56\ v^{0.466}$.

Then, we carry out the calculations of the mathematical model (I.4),(I.8) for all new values of the coefficients at air-cooling velocities v=0.4, v=1.2 and v=2.4m/s consecutively. The results are shown in Fig.5. Two expected regularities take place: the increase of the cooling surface area leads to the decrease of surface gas temperature $T_{w3}$ and of the highest gas temperature $T_{max}$ in the cross-section of the laser tube; on the other hand the ratio $\eta$ increases. In view of the bigger cooling surface area, the rates of reduction of $T_{max}$ and $T_{w3}$ are more important than in Fig.2.

For all considered cases we also increase the rate of applied electric power P in order to achieve the initial optimal temperature $T_{max}$, which is valid for natural convection (v=0). The numerical procedure is the same as in the previous section. The obtained results are shown in Fig.6.

Nevertheless of the fact that in the cross-section of the laser tube the highest gas temperature $T_{max}$ is achieved, the surface temperatures $T_{w3}$ for four different values of v are not equal. Then, the temperature profiles in the cross-section of the tube are also different.

As in the previous case without additional metal fins, the increase of the cooling velocity v is naturally limited, because of the possible condensation of the Cd-vapor on the walls of the reservoir. Comparing the rates of surface temperatures $T_{w3}$ in Fig.4 and Fig.7 we can see that the restriction for air-cooling velocity at surface area extension by rectangular fins is more significant, than without fins.

## Conclusions.

The proposed mathematical model of heat exchange (I.4), (I.8) allows to find the gas temperature of the RF-excited gas discharge in laser tube under various boundary-value conditions. So, by means of numerical simulations we can predict the rates of possible increase of the applied electric power P at a given optimal temperature distribution.

Taking into account the obtained results we arrive at the following conclusion: The extension of a cooling surface area allows to apply lower velocities of the airflows so that the operation exploitation of the laser will be more efficient. Another conclusion with practical importance is the possibility to increase the applied electric power up to 35-40%.

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