THE SENSE OF THE SUCCESSFUL MATHEMATICS TEACHER

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ABSTRACT

The article suggests an idea for the formation of a very specific state of Mind, Feeling, Taste or Sense of Mathematics that appears in the course of time with teachers who can be defined as successful in the very field of pedagogical practices. This phenomenon and its importance for the upbringing, education and the successful performance of both students and teachers are discussed and revealed on the basis of mathematical problem solving in the 5th to 12th grades of secondary schools.

When a Mathematics teacher succeeds in achieving all set goals during the course of a lesson at school, both he/she and students are pleased with their attending the lesson. In that situation, we can state the terms of the successful teacher. Otherwise, the very constitution of the term "successful teacher" is invalid.

From our point of view, a necessary condition for a teacher to become worthy of being considered a successful one is related to the existence of a kind of a very special "feeling", "instinct", "taste", "sensitivity" or "sense" of Mathematics in his/her pedagogical practice. For short, further we shall call it "Sense of mathematics (sensitivity)" connected to the concept of mathematical sense. It is well known, that the acceptance of the Mathematics science is comparable to the acceptance and understanding of art. Furthermore, if the understanding of a musical piece or art painting is hard and not obtained by everyone, then understanding of pure Mathematics is stated to be even much harder and complicated. The reason for that is the nature of the Mathematical science as an abstract and highly artistic state of mind. Connection to that level of art can be achieved with the help of the term "sense of mathematics".

What does "mathematics feeling or sense" mean? The sense of mathematics lies in the core foundations of understanding how the mind operates. The sense of mathematics is used as an art to express the process of refreshing the obtained decision and as such is supplied by discoveries, tricks and wits. The term is based on the human need for rationalization of everything we do and therefore is closely related with the basics of systematic study, so it is a product of the mental process of realizing and figuring out. That is a creative moment inspired by the inner awareness of both student and teacher and exceeds to a quality expressed manner of one's teaching. In that manner of speaking, the mathematical sense opposes all kinds of formality in teachers' work. It states affection of one's work and a highly developed taste towards creativity and inspiration. Understanding the mathematical sense as an inspiration spark, one is able to reach out for the heights of pure creation away from templated ways of common thinking. The mathematical sense is an art by means of which we are able to acquire ideas and knowledge. It states a classic form of emanation towards skills and as such grows, matures and develops. In the spirit of all said above, the mathematical sense is to be nurtured at a very early age by problems with multiple solutions and deferential comparison of all discovered solutions. The author's mathematical sense to recognize the decision at the precise moment of discovering solutions is at its best. The same thing also happens in the process of figuring out how one problem has been created. Growing and developing such a sense is possible in many different ways. For example, through research.

We are offering the reader a list of mathematical problems that are given to pupils from elementary to graduate school. Let us start with the idea of rational calculations.

Problem 1. Find the sum through rational calculations:

a) $1-2+3-4+5-6$;
b) $2+3+8-1-5-8-11$;
c) $-98+25-2+75$;
d) $1-\frac{1}{2}-\frac{1}{3}+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+\frac{1}{4}$

Problem 2. Find the sum of:

$$S=1+2+2^2+...+2^{2008}$$

Solution:

$$2.S=2+2^2+2^3+...+2^{2009}$$
$$1+2.S=1+2+2^2+...+2^{2008}+2^{2009}$$
$$1+2.S=S+2^{2009}$$
$$S=2^{2009}-1$$

Problem 3: Calculate: $19871987.198919891989 - 19891989.198719871987$

Solution: It is obvious that

$$19871987 = 1987.10001$$
$$198919891989 = 1989.100010001$$
$$19891989 = 1989.10001$$
$$198719871987 = 1987.100010001$$

Therefore after subtraction the result is zero.

Problem 4: Calculate:

$$\frac{(1986^2 - 1992).(1986^2 + 3972 - 3).1987}{1983.1985.1988.1989}$$

Problem 5: Calculate:

$$N = \frac{1234567890}{1234567891^2 - 1234567890.1234567892}$$

Problem 6: Expand to simple multipliers

$$989.1001.1007 + 320$$

One of the ways to point out the presence of mathematical sense is through the ability of "expanding" and transferring one difficult problem into several much easier ones. The ability to summarize one problem and point out the similarities in the differences is another important trait. Here it is visible to see the ability to discover similarities and analogy where they can be least expected and still to be able to generate ideas and decisions as results of mind-crafted exercises.

Forming and shaping of mathematical sense (or "feeling") is achieved also through the idea of a multi-decisional problem. That is mostly understood with the solution of a problem in many different ways. Much more successful is its appearance in one's collision with a difficult problem, where even ONE SINGLE correct answer and solution may be considered a success and anything more than that say, FIVE, - even a TRIUMPH.

Let us see how that applies in practice:

Problem 7. Which one is bigger than the other: $\sqrt[3]{2}+\sqrt{3}$ or 3?

Solution: Let us assume that $\sqrt[3]{2}+\sqrt{3}>3$. Then

$$\sqrt[3]{2} > 3-\sqrt{3}$$
$$2=(\sqrt[3]{2})^3 >$$
$$(3-\sqrt{3})^3=27-27.\sqrt{3}+27-3.\sqrt{3}=54-30.\sqrt{3}$$
$$30.\sqrt{3} > 52$$
$$15.\sqrt{3} > 26$$

By raising by the square the last inequality, one gets $675>676$, that is pure contradiction.
Therefore, the assumption is incorrect and because of that it is true looked in the opposite direction.

Solution 2: Having in mind that $\sqrt{3}=1,73...\angle\frac{174}{100}=\frac{87}{50}$, let us prove that $\sqrt[3]{2}\angle\frac{63}{50}$ is also true. We know that

$(50.\sqrt[3]{2})^3=2.50.50.50=250000\angle 250047=63^3$ is true.

Therefore $\sqrt[3]{2}+\sqrt{3}$ will not exceed the amount of $\frac{87}{50}+\frac{63}{50}=\frac{150}{50}=3$.

By that the inequality is proven.

Solution 3: We notice that $\sqrt[3]{2}-1, 2-\sqrt{3}$ are positive and also that

$$(\sqrt[3]{2}-1).(\sqrt[3]{4}+\sqrt[3]{2}+1)=1=(2-\sqrt{3}).(2+\sqrt{3})$$

Showing that $\sqrt[3]{4}+\sqrt[3]{2}+1$ is greater than $2+\sqrt{3}$, from the statement above will follow that $2-\sqrt{3}$ is greater than $\sqrt[3]{2}-1$ and therefore the wanted inequality is proven.

Using the average arithmetical and average geometrical inequality of two figures, we get:

$$\sqrt[3]{4}+\sqrt[3]{2} \; > \; 2.\sqrt{2} = 2.\sqrt{\sqrt[3]{4}.\sqrt[2]{2}}$$
$$\sqrt[3]{4}+\sqrt[3]{2} \; +1 > 2.\sqrt{2}+1$$

Now, we shall prove that $2.\sqrt{2}+1 \; > \; 2+\sqrt{3}$. From $2>\sqrt{3}$ we get that $4>2.\sqrt{3}$

$$4 \; > \; 2.\sqrt{3}$$
$$8 \; > \; 1+2\sqrt{3}+3=(1+\sqrt{3})^2$$

When we get the square root of

$$2\sqrt{2} \; > 1+\sqrt{3}$$
$$2\sqrt{2}+1 > \; 2+\sqrt{3}$$

we also get the proof to be completed.

Solution 4. Let us assume that $\sqrt[3]{2}-1 \; > \; 2-\sqrt{3}$. We multiply both sides of the inequality with $2+\sqrt{3}$ and we get

$$(2+\sqrt{3}).(\sqrt[3]{2}-1) \; > 1=(\sqrt[3]{2}-1).(\sqrt[3]{4}+\sqrt[3]{2}+1)$$

Dividing the inequality by $\sqrt[3]{2}-1$ we get

$$2+\sqrt{3} \; > \; \sqrt[3]{4}+\sqrt[3]{2} \; +1$$
$$\sqrt{3} \; > \; \sqrt[3]{4}+\sqrt[3]{2} \; -1$$

After raising both sides of the last inequality to the power of two, we get

$$3 \; > \; 5-\sqrt[3]{4}$$
$$\sqrt[3]{4} \; > 2$$

When we raise both sides of the inequality to the power of three, we get the ridiculous inequality 4 >8. Therefore $2-\sqrt{3} \; > \; \sqrt[3]{2}-1$ the wanted proof is found.

Solution 5: Let $a=3-\sqrt[3]{2}$ Then

$$3-a=\sqrt[2]{2}$$
$$27-27a+9a^2-a^3=2$$
$$a^3-9a^2+27a-25=0$$

That states that **a** is a root of $f(x) = x^3 - 9x^2 + 27x - 25 = 0$. It is obvious that $f'(x) = 3x^2 - 18x + 27 = 3.(x-3)^2 \geq 0$, f(x) is an increasing function. Knowing that $f(\sqrt{3}) = 3\sqrt{3} - 27 + 27\sqrt{3} - 25 = 2(15\sqrt{3} - 26) = -\frac{2}{15\sqrt{3}+26}$ has a negative value, it follows that $f(a) = 0 > f(\sqrt{3})$ and since f(x) is an increasing function, we get that $a > \sqrt{3}$. In other words, the required inequality is proven.

That special "feeling", or "sense" of Mathematics we are talking about obviously arises and develops with those who have some inner need or predisposition towards perfection. That suggests and requires before all the ever growing desire of learning and not settling down for the most obvious solution and avoiding the toughest problems. That process is inseparable from the desire of experimenting and that process definitely includes creating own problems, comparing, evaluating and imitating others.

The appearance and existence of such a "feeling" or "sense" with the teacher is quintessential for one's experience, one's intuition to define / recognize and see in perspective what each and every student requires and how to nurture students' gifts. That is what gives the teacher the status of a certain destiny creator. It is this sense and no other thing that could be compared with the set of knowledge thirst in student's soul and keeping up the wave. All that is placed in the process of choosing the most suitable problems and their graduation, and also in the way of suggesting non - standard solution (answer) of the hardest problems. That is how the mathematical sense of the teacher evolves in the art of taste creation and mind style formation.

To be able to really encourage students, a teacher should have a true sense (feeling, taste or touch) of things that includes a kind of an effective state to feel emotional linkage as well. Mathematical sense discovers also the beauty of the mathematical art, shaping it into an aesthetical form of problem and solution.

For the successful Mathematics teacher, the well guided sense is shown in every lesson at school, as well as the preparation for the lesson itself. There can be pointed out also the ability to feel and pick up the right moment to take chance of heuristics, search of ideas and overcoming obstacles. The successful teacher's sense takes part while picking up problems and themes, stating questions about the matter of ideas, the art of thinking itself and the experience at large. That special sensing can be felt also not only in the process of questioning students' knowledge and skills, but also in the discovery of students' mathematical talents and once found – in the process of developing and encouraging them. For the successful teacher, the mathematical sense is crucial when the teacher should assist his/her student to overcome mathematical obstacles. A teacher's sense is a means for a new beginning about updating teaching practices, pedagogical ways and opportunities of development.

For the successful teacher, the mathematical sense registers the readiness to accept certain truths and ways of thinking. That sense gives opportunities for creativity, development, success and career. It also gives the teacher certain special inner “markers” and highlights that enable him/her to timely figure our certain very important and even crucial points of both the teaching and the learning process. The mathematical sense has an imaginary effect on the association process and the ways of thinking. All that is connected to quick wit, tricks and calculations that encourage teachers to seek for new paths in the same logical scheme and system. Sense is to be able to find opportunities where others see obstacles and difficulties. It requires distance from the common trends and fashions and seeking for unusual paths. That alone constitutes further promoting the patterns and appeals for creative inspiration and as such is a step into another dimension of the teacher’s practice by means of interpretation and organization.

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