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TRANSFORMATION OF SPACES OF COMPOSITIONS

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Abstract. In the present paper three new connections ${}^1\Gamma^{\sigma}_{\alpha\beta}$, ${}^2\Gamma^{\sigma}_{\alpha\beta}$ and ${}^3\Gamma^{\sigma}_{\alpha\beta} = \frac{1}{2}({}^1\Gamma^{\sigma}_{\alpha\beta} +{}^2 \Gamma^{\sigma}_{\alpha\beta})$ are introduced to any weyl connection $\Gamma^{\sigma}_{\alpha\beta}$ with the help of the projecting affinors of a composition. The spaces of compositions W_N, ${}^1A_n$, ${}^2A_n$, ${}^3A_n$ with connections $\Gamma^{\sigma}_{\alpha\beta}$, ${}^1\Gamma^{\sigma}_{\alpha\beta}$, ${}^2\Gamma^{\sigma}_{\alpha\beta}$, ${}^3\Gamma^{\sigma}_{\alpha\beta}$, respectively are considered.

It is proved that if one of these spaces has special composition $X_n \times X_m (n + m = N)$ then and the rest three spaces are spaces of the same special composition.

Key words: Weyl space, spaces of compositions, affinors of compositions
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1. Preliminary

Let the field of the affinor a^{β}_{α} satisfying the condition [1], [2]

$$a^{\sigma}_{\alpha} a^{\beta}_{\sigma} = \delta^{\beta}_{\alpha}. \tag{1}$$

be given in the differentiable manifold X_N.

Let an arbitrary symmetric connection $\Gamma^{\sigma}_{\alpha\beta}$ be given in X_N. The differentiable manifold X_N supplied with the connection $\Gamma^{\sigma}_{\alpha\beta}$ will be denoted by A_N.

The giving of the field of the affinor a^{β}_{α} satisfying the condition (1) is equivalent to the giving of the composition $X_n \times X_m (n + m = N)$ in A_N [1].

The affinor a^{β}_{α} is called an affinor of the composition. Two positions $P(X_n)$ and $P(X_m)$ of the base manifolds pass through any point of the space $A_N(X_n \times X_m)$.

According to [1], [2] the condition for integrability of the structure characterizes with the equality

$$a^{\sigma}_{\beta}\nabla_{[\alpha}\ a^{\nu}_{\sigma]} - a^{\sigma}_{\alpha}\nabla_{[\beta}\ a^{\nu}_{\sigma]} = 0. \tag{2}$$

The projecting affinors $\overset{n}{a}{}^{\beta}_{\alpha}$ *and* $\overset{m}{a}{}^{\beta}_{\alpha}$, define by the equalities [4]

$$\overset{n}{a}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^{\beta}_{\alpha} + a^{\beta}_{\alpha})\ ,\quad \overset{m}{a}{}^{\beta}_{\alpha} = \frac{1}{2}(\delta^{\beta}_{\alpha} - a^{\beta}_{\alpha})\ , \tag{3}$$

satisfy the following conditions [3], [4]:

$$\begin{array}{lll} \overset{n}{a}{}^{\beta}_{\alpha} + \overset{m}{a}{}^{\beta}_{\alpha} = \delta^{\beta}_{\alpha}\ , & \overset{n}{a}{}^{\beta}_{\alpha} - \overset{m}{a}{}^{\beta}_{\alpha} = a^{\beta}_{\alpha}\ , & \\ \overset{n}{a}{}^{\beta}_{\alpha}\overset{n}{a}{}^{\gamma}_{\beta} = \overset{n}{a}{}^{\gamma}_{\alpha}\ , & \overset{m}{a}{}^{\beta}_{\alpha}\overset{m}{a}{}^{\gamma}_{\beta} = \overset{m}{a}{}^{\gamma}_{\alpha}\ , & \overset{m}{a}{}^{\beta}_{\alpha}\overset{n}{a}{}^{\gamma}_{\beta} = 0\ , \\ \overset{n}{a}{}^{\beta}_{\alpha}\ a^{\gamma}_{\beta} = a^{\beta}_{\alpha}\ \overset{n}{a}{}^{\gamma}_{\beta} = \overset{n}{a}{}^{\gamma}_{\alpha}\ , & \overset{m}{a}{}^{\beta}_{\alpha}\ a^{\gamma}_{\beta} = a^{\beta}_{\alpha}\ \overset{m}{a}{}^{\gamma}_{\beta} = -\overset{m}{a}{}^{\gamma}_{\alpha}\ . & \end{array} \tag{4}$$

Any vector $v^{\alpha} \in A_N$ has the following representation [3]

$$v^{\alpha} = \overset{n}{a}{}^{\alpha}_{\sigma}v^{\sigma} + \overset{m}{a}{}^{\alpha}_{\sigma}v^{\sigma} = \overset{n}{V}{}^{\alpha} + \overset{m}{V}{}^{\alpha}, \tag{5}$$

where $\overset{n}{V}{}^{\alpha} = \overset{n}{a}{}^{\alpha}_{\sigma}v^{\sigma} \in P(X_n)$, $\overset{m}{V}{}^{\alpha} = \overset{m}{a}{}^{\alpha}_{\sigma}v^{\sigma} \in P(X_m)$.

Characteristics of the following compositions are obtained in [2]:

1. $(d-d)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along any line of the space characterize with the condition

$$\nabla_{\sigma}a^{\beta}_{\alpha} = 0. \tag{6}$$

2. $(ch-ch)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along X_m and X_n, respectively characterize with the condition

$$\nabla_{[\sigma}a^{\beta}_{\alpha]} = 0. \tag{7}$$

3. $(g-g)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along X_n and X_m, respectively characterize with the condition

$$a^{\sigma}_{\beta}\ \nabla_{\alpha}a^{\nu}_{\sigma} + a^{\sigma}_{\alpha}\ \nabla_{\sigma}a^{\nu}_{\beta} = 0. \tag{8}$$

4. $(g - ch)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along X_n characterize with the condition

$$\overset{n}{a}{}^{\sigma}_{\alpha} \nabla_\sigma \overset{n}{a}{}^{\nu}_{\beta} = 0. \tag{9}$$

5. $(ch - g)$-compositions for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along X_m characterize with the condition

$$\overset{m}{a}{}^{\sigma}_{\alpha} \nabla_\sigma \overset{m}{a}{}^{\nu}_{\beta} = 0. \tag{10}$$

6. $(X_n - d)$-compositions (resp. $(d - X_m)$-compositions) for which the positions $P(X_m)$ (resp. $P(X_n)$) are parallelly translated along any line of the space characterize with the condition

$$\overset{m}{a}{}^{\sigma}_{\alpha} \nabla_\sigma \overset{n}{a}{}^{\nu}_{\beta} = 0 \quad \left(resp. \quad \overset{n}{a}{}^{\sigma}_{\alpha} \nabla_\sigma \overset{m}{a}{}^{\nu}_{\beta} = 0\right). \tag{11}$$

7. $(X_n - ch)$-compositions (resp. $(ch - X_m)$-compositions) for which the positions $P(X_m)$ (resp. $P(X_n)$) are parallelly translated along $P(X_n)$ (resp. $P(X_m)$) characterize with the condition

$$\overset{n}{a}{}^{\sigma}_{\alpha} \overset{m}{a}{}^{\nu}_{\beta} \nabla_\sigma \overset{m}{a}{}^{\beta}_{\nu} = 0 \quad \left(resp. \quad \overset{m}{a}{}^{\sigma}_{\alpha} \overset{n}{a}{}^{\nu}_{\beta} \nabla_\sigma \overset{n}{a}{}^{\beta}_{\nu} = 0\right). \tag{12}$$

8. $(X_n - g)$-compositions (resp. $(g - X_m)$-compositions) for which the positions $P(X_m)$ (resp. $P(X_n)$) are parallelly translated along $P(X_m)$ (resp. $P(X_n)$) characterize with the condition

$$\overset{m}{a}{}^{\sigma}_{\alpha} \overset{m}{a}{}^{\nu}_{\beta} \nabla_\sigma \overset{m}{a}{}^{\beta}_{\nu} = 0 \quad \left(resp. \quad \overset{n}{a}{}^{\sigma}_{\alpha} \overset{n}{a}{}^{\nu}_{\beta} \nabla_\sigma \overset{n}{a}{}^{\beta}_{\nu} = 0\right). \tag{13}$$

2. Transformation of spaces of compositions

Let the weyl connection $\Gamma^\sigma_{\alpha\beta}$ define with symmetric tensor $g_{\alpha\beta}$ and covector ω_σ so that $\nabla_\sigma g_{\alpha\beta} = 2\omega_\sigma g_{\alpha\beta}$ be given in the differentiable manifold X_N with an affinor a^β_α, satisfying the condition (1). The N-dimensional weyl space with a fundamental tensor $g_{\alpha\beta}$ and complementery covector ω_σ will be denoted W_N.

Consider the following symmetric connections:

$${}^1\Gamma^\sigma_{\alpha\beta} = \Gamma^\sigma_{\alpha\beta} + \overset{n}{a}{}^{\sigma}_{\alpha} \overset{n}{\omega}_\beta + \overset{n}{a}{}^{\sigma}_{\beta} \overset{n}{\omega}_\alpha, \tag{14}$$

$${}^2\Gamma^\sigma_{\alpha\beta} = \Gamma^\sigma_{\alpha\beta} + \overset{m}{a}{}^{\sigma}_{\alpha} \overset{m}{\omega}_\beta + \overset{m}{a}{}^{\sigma}_{\beta} \overset{m}{\omega}_\alpha, \tag{15}$$

where

$$\overset{n}{\omega}_\beta = \overset{n}{a}{}^{\sigma}_{\beta}\,\omega_\sigma\ ,\quad \overset{m}{\omega}_\beta = \overset{m}{a}{}^{\sigma}_{\beta}\,\omega_\sigma. \tag{16}$$

The mean connection for connections ${}^1\Gamma^\sigma_{\alpha\beta}$ and ${}^2\Gamma^\sigma_{\alpha\beta}$ will be denoted ${}^3\Gamma^\sigma_{\alpha\beta}$ i.e. ${}^3\Gamma^\sigma_{\alpha\beta} = \dfrac{1}{2}({}^1\Gamma^\sigma_{\alpha\beta} + {}^2\,\Gamma^\sigma_{\alpha\beta})$.

Denote by ${}^1A_N,\ {}^2A_N,\ {}^3A_N$ the space X_N supplied with the connections ${}^1\Gamma^\sigma_{\alpha\beta}$, ${}^2\Gamma^\sigma_{\alpha\beta}$, ${}^3\Gamma^\sigma_{\alpha\beta}$, respectively. Let $\nabla,\ {}^1\nabla,\ {}^2\nabla,\ {}^3\nabla$ be the covariant derivatives in the spaces $W_N,\ {}^1A_N,\ {}^2A_N,\ {}^3A_N$, respectively. From (1) it follows that $W_N,\ {}^1A_N,\ {}^2A_N,\ {}^3A_N$ are spaces of compositions $X_n \times X_m (n+m=N)$.

Lemma 1. *The covariant derivatives of the affinor a^β_α and projecting affinors $\overset{n}{a}{}^{\beta}_{\alpha}$, $\overset{m}{a}{}^{\beta}_{\alpha}$ satisfy the following equalities:*

$$\nabla_\sigma a^\beta_\alpha = {}^1\nabla_\sigma a^\beta_\alpha = {}^2\nabla_\sigma a^\beta_\alpha = {}^3\nabla_\sigma a^\beta_\alpha\ , \tag{17}$$

$$\nabla_\sigma \overset{n}{a}{}^{\beta}_{\alpha} = {}^1\nabla_\sigma \overset{n}{a}{}^{\beta}_{\alpha} = {}^2\nabla_\sigma \overset{n}{a}{}^{\beta}_{\alpha} = {}^3\nabla_\sigma \overset{n}{a}{}^{\beta}_{\alpha}\ , \tag{18}$$

$$\nabla_\sigma \overset{m}{a}{}^{\beta}_{\alpha} = {}^1\nabla_\sigma \overset{m}{a}{}^{\beta}_{\alpha} = {}^2\nabla_\sigma \overset{m}{a}{}^{\beta}_{\alpha} = {}^3\nabla_\sigma \overset{m}{a}{}^{\beta}_{\alpha}\ . \tag{19}$$

Proof. Using (14) we obtain

$${}^1\nabla_\sigma a^\beta_\alpha - \nabla_\sigma a^\beta_\alpha = (\overset{n}{a}{}^{\beta}_{\sigma}\,\overset{n}{\omega}_\nu + \overset{n}{a}{}^{\beta}_{\nu}\,\overset{n}{\omega}_\sigma)a^\nu_\alpha - (\overset{n}{a}{}^{\nu}_{\sigma}\,\overset{n}{\omega}_\alpha + \overset{n}{a}{}^{\nu}_{\alpha}\,\overset{n}{\omega}_\sigma)a^\beta_\nu,$$

from where taking into account (4) we establish

$${}^1\nabla_\sigma a^\beta_\alpha - \nabla_\sigma a^\beta_\alpha = \overset{n}{a}{}^{\beta}_{\sigma}(a^\nu_\alpha\,\overset{n}{\omega}_\nu - \overset{n}{\omega}_\alpha) = \overset{n}{a}{}^{\beta}_{\sigma}[(\overset{n}{a}{}^{\nu}_{\alpha} - \overset{m}{a}{}^{\nu}_{\alpha})\overset{n}{\omega}_\nu - \overset{n}{\omega}_\alpha].$$

Now from (4) and (16) it follows

$${}^1\nabla_\sigma a^\beta_\alpha - \nabla_\sigma a^\beta_\alpha = \overset{n}{a}{}^{\beta}_{\sigma}[(\overset{n}{a}{}^{\nu}_{\alpha} - \overset{m}{a}{}^{\nu}_{\alpha})\overset{n}{a}{}^{\rho}_{\nu}\,\omega_\rho - \overset{n}{\omega}_\alpha] = \overset{n}{a}{}^{\beta}_{\sigma}(\overset{n}{a}{}^{\rho}_{\alpha}\omega_\rho - \overset{n}{\omega}_\alpha) =$$
$$\overset{n}{a}{}^{\beta}_{\sigma}(\overset{n}{\omega}_\alpha - \overset{n}{\omega}_\alpha) = 0, \quad \text{i.e.}\quad {}^1\nabla_\sigma a^\beta_\alpha = \nabla_\sigma a^\beta_\alpha.$$

Using (15) we obtain

$${}^2\nabla_\sigma a^\beta_\alpha - \nabla_\sigma a^\beta_\alpha = (\overset{m}{a}{}^{\beta}_{\sigma}\,\overset{m}{\omega}_\nu + \overset{m}{a}{}^{\beta}_{\nu}\,\overset{m}{\omega}_\sigma)a^\nu_\alpha - (\overset{m}{a}{}^{\nu}_{\sigma}\,\overset{m}{\omega}_\alpha + \overset{m}{a}{}^{\nu}_{\alpha}\,\overset{m}{\omega}_\sigma)a^\beta_\nu,$$

from where taking into account (4) and (16) we establish

$${}^2\nabla_\sigma a^\beta_\alpha - \nabla_\sigma a^\beta_\alpha = \overset{m}{a}{}^{\beta}_{\sigma}(a^\nu_\alpha\,\overset{m}{\omega}_\nu + \overset{m}{\omega}_\alpha) = \overset{m}{a}{}^{\beta}_{\sigma}[(\overset{n}{a}{}^{\nu}_{\alpha} - \overset{m}{a}{}^{\nu}_{\alpha})\overset{m}{a}{}^{\rho}_{\nu}\,\omega_\rho + \overset{m}{\omega}_\alpha] =$$
$$\overset{m}{a}{}^{\beta}_{\sigma}(-\overset{m}{a}{}^{\rho}_{\alpha}\,\omega_\rho + \overset{m}{\omega}_\alpha) = \overset{m}{a}{}^{\beta}_{\sigma}(-\overset{m}{\omega}_\alpha + \overset{n}{\omega}_\alpha) = 0, \quad \text{i.e.}\quad {}^2\nabla_\sigma a^\beta_\alpha = \nabla_\sigma a^\beta_\alpha.$$

From ${}^3\Gamma^\sigma_{\alpha\beta} = \dfrac{1}{2}({}^1\Gamma^\sigma_{\alpha\beta} + {}^2\Gamma^\sigma_{\alpha\beta})$ it follows ${}^3\nabla_\sigma a^\beta_\alpha = \dfrac{1}{2}({}^1\nabla_\sigma a^\beta_\alpha + {}^2\nabla_\sigma a^\beta_\alpha) = \nabla_\sigma a^\beta_\alpha$.
Thus we proved the equalities (17).

According to (14) we have

$$ {}^1\nabla_\sigma \overset{n}{a}{}^\beta_\alpha - \nabla_\sigma \overset{n}{a}{}^\beta_\alpha = (\overset{n}{a}{}^\beta_\sigma \overset{n}{\omega}_\nu + \overset{n}{a}{}^\beta_\nu \overset{n}{\omega}_\sigma)\overset{n}{a}{}^\nu_\alpha - (\overset{n}{a}{}^\nu_\sigma \overset{n}{\omega}_\alpha + \overset{n}{a}{}^\nu_\alpha \overset{n}{\omega}_\sigma)\overset{n}{a}{}^\beta_\nu, $$

from where taking into account (4) and (16) we obtain

$$ {}^1\nabla_\sigma \overset{n}{a}{}^\beta_\alpha - \nabla_\sigma \overset{n}{a}{}^\beta_\alpha = \overset{n}{a}{}^\beta_\sigma(\overset{n}{a}{}^\nu_\alpha \overset{n}{\omega}_\nu - \overset{n}{\omega}_\alpha) = \overset{n}{a}{}^\beta_\sigma(\overset{n}{a}{}^\nu_\alpha \overset{n}{a}{}^\rho_\nu\, \omega_\rho - \overset{n}{\omega}_\alpha) = $$

$$ \overset{n}{a}{}^\beta_\sigma(\overset{n}{a}{}^\rho_\alpha\, \omega_\rho - \overset{n}{\omega}_\alpha) = \overset{n}{a}{}^\beta_\sigma(\overset{n}{\omega}_\alpha - \overset{n}{\omega}_\alpha) = 0, \quad \text{i.e.} \quad {}^1\nabla_\sigma \overset{n}{a}{}^\beta_\alpha = \nabla_\sigma \overset{n}{a}{}^\beta_\alpha. $$

According to (15) we can write

$$ {}^2\nabla_\sigma \overset{n}{a}{}^\beta_\alpha - \nabla_\sigma \overset{n}{a}{}^\beta_\alpha = (\overset{m}{a}{}^\beta_\sigma \overset{m}{\omega}_\nu + \overset{m}{a}{}^\beta_\nu \overset{m}{\omega}_\sigma)\overset{n}{a}{}^\nu_\alpha - (\overset{m}{a}{}^\nu_\sigma \overset{m}{\omega}_\alpha + \overset{m}{a}{}^\nu_\alpha \overset{m}{\omega}_\sigma)\overset{n}{a}{}^\beta_\nu, $$

from where taking into account (4) and (16) we obtain

$$ {}^2\nabla_\sigma \overset{n}{a}{}^\beta_\alpha - \nabla_\sigma \overset{n}{a}{}^\beta_\alpha = \overset{m}{a}{}^\beta_\sigma \overset{n}{a}{}^\nu_\alpha \overset{m}{\omega}_\nu = \overset{m}{a}{}^\beta_\sigma \overset{n}{a}{}^\nu_\alpha \overset{m}{a}{}^\rho_\nu\, \omega_\rho = 0 \ \text{ i.e. } \ {}^2\nabla_\sigma \overset{n}{a}{}^\beta_\alpha = \nabla_\sigma \overset{n}{a}{}^\beta_\alpha. $$

Hence ${}^3\nabla_\sigma \overset{n}{a}{}^\beta_\alpha = \dfrac{1}{2}({}^1\nabla_\sigma \overset{n}{a}{}^\beta_\alpha + {}^2\nabla_\sigma \overset{n}{a}{}^\beta_\alpha) = \nabla_\sigma \overset{n}{a}{}^\beta_\alpha$.
Thus we proved the and equalities (18).

According to (14) we have

$$ {}^1\nabla_\sigma \overset{m}{a}{}^\beta_\alpha - \nabla_\sigma \overset{m}{a}{}^\beta_\alpha = (\overset{n}{a}{}^\beta_\sigma \overset{n}{\omega}_\nu + \overset{n}{a}{}^\beta_\nu \overset{n}{\omega}_\sigma)\overset{m}{a}{}^\nu_\alpha - (\overset{n}{a}{}^\nu_\sigma \overset{n}{\omega}_\alpha + \overset{n}{a}{}^\nu_\alpha \overset{n}{\omega}_\sigma)\overset{m}{a}{}^\beta_\nu, $$

from where taking into account (4) and (16) we obtain

$$ {}^1\nabla_\sigma \overset{m}{a}{}^\beta_\alpha - \nabla_\sigma \overset{m}{a}{}^\beta_\alpha = \overset{n}{a}{}^\beta_\sigma \overset{m}{a}{}^\nu_\alpha \overset{n}{\omega}_\nu = \overset{n}{a}{}^\beta_\sigma \overset{m}{a}{}^\nu_\alpha \overset{n}{a}{}^\rho_\nu\, \omega_\rho = 0 \ \text{ i.e. } \ {}^1\nabla_\sigma \overset{m}{a}{}^\beta_\alpha = \nabla_\sigma \overset{m}{a}{}^\beta_\alpha. $$

According to (15) we can write

$$ {}^2\nabla_\sigma \overset{m}{a}{}^\beta_\alpha - \nabla_\sigma \overset{m}{a}{}^\beta_\alpha = (\overset{m}{a}{}^\beta_\sigma \overset{m}{\omega}_\nu + \overset{m}{a}{}^\beta_\nu \overset{m}{\omega}_\sigma)\overset{m}{a}{}^\nu_\alpha - (\overset{m}{a}{}^\nu_\sigma \overset{m}{\omega}_\alpha + \overset{m}{a}{}^\nu_\alpha \overset{m}{\omega}_\sigma)\overset{m}{a}{}^\beta_\nu, $$

from where taking into account (4) and (16) we obtain

$$ {}^2\nabla_\sigma \overset{m}{a}{}^\beta_\alpha - \nabla_\sigma \overset{m}{a}{}^\beta_\alpha = \overset{m}{a}{}^\beta_\sigma(\overset{m}{a}{}^\nu_\alpha \overset{m}{\omega}_\nu - \overset{m}{\omega}_\alpha) = \overset{m}{a}{}^\beta_\sigma(\overset{m}{a}{}^\nu_\alpha \overset{m}{a}{}^\rho_\nu\, \omega_\rho - \overset{m}{\omega}_\alpha) = $$

$$ \overset{m}{a}{}^\beta_\sigma(\overset{m}{a}{}^\rho_\alpha\, \omega_\rho - \overset{m}{\omega}_\alpha) = \overset{m}{a}{}^\beta_\sigma(\overset{m}{\omega}_\alpha - \overset{m}{\omega}_\alpha) = 0, \quad \text{i.e.} \quad {}^2\nabla_\sigma \overset{m}{a}{}^\beta_\alpha = \nabla_\sigma \overset{m}{a}{}^\beta_\alpha. $$

Hence ${}^3\nabla_\sigma \overset{m}{a}{}^\beta_\alpha = \dfrac{1}{2}({}^1\nabla_\sigma \overset{m}{a}{}^\beta_\alpha + {}^2\nabla_\sigma \overset{m}{a}{}^\beta_\alpha) = \nabla_\sigma \overset{m}{a}{}^\beta_\alpha$

Thus we proved the equalities (19) and the lemma. □

Theorem 1. *If one of the spaces* $W_N,\ {}^1A_N,\ {}^2A_N,\ {}^3A_N$ *is a space of the special composition* $(d-d)$ *or* $(ch-ch)$, *or* $(g-g)$, *or* $(g-ch)$, *or* $(ch-g)$, *or* (X_m-d), *or* $(d-X_m)$, *or* (X_m-g), *or* $(g-X_m)$, *or* (X_m-ch), *or* $(ch-X_m)$ *then the rest three spaces are spaces of the same special composition.*

Proof. From the condition for integrability (2) and from the equality (17) of the lemma it follows that if the structure of one of the spaces of compositions $W_N,\ {}^1A_N,\ {}^2A_N,\ {}^3A_N$ is integrable, then the structures of the rest three spaces are integribale too.

From the characteristics (6), (7), (8), (9), (10), (11), (12), (13) of the special compositions and by the lemma it follows that if the composition $X_n \times X_m$ is special from the kind $(d-d)$ or $(ch-ch)$, or $(g-g)$, or $(g-ch)$, or $(ch-g)$, or (X_m-d), or $(d-X_m)$, or (X_m-g), or $(g-X_m)$, or (X_m-ch), or $(ch-X_m)$ in one of the spaces $W_N,\ {}^1A_N,\ {}^2A_N,\ {}^3A_N$, then this composition is special of the same kind and in the rest three spaces.

Thus the theorem is proved. □

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ПРЕОБРАЗУВАНЕ НА ПРОСТРАНСТВА ОТ КОМПОЗИЦИИ

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Резюме. В настоящата работа са въведени към произволна вайлова свързаност $\Gamma^{\sigma}_{\alpha\beta}$ три нови свързаности ${}^1\Gamma^{\sigma}_{\alpha\beta}$, ${}^2\Gamma^{\sigma}_{\alpha\beta}$ and ${}^3\Gamma^{\sigma}_{\alpha\beta} = \frac{1}{2}({}^1\Gamma^{\sigma}_{\alpha\beta}+{}^2\Gamma^{\sigma}_{\alpha\beta})$ с помощта на проектиращите афинори на композиция.

Разгледани са пространства от композиции W_N, ${}^1A_n$, ${}^2A_n$, ${}^3A_n$ съответно със следните свързаности $\Gamma^{\sigma}_{\alpha\beta}$, ${}^1\Gamma^{\sigma}_{\alpha\beta}$, ${}^2\Gamma^{\sigma}_{\alpha\beta}$, ${}^3\Gamma^{\sigma}_{\alpha\beta}$.

Доказано е, че ако едно от тези пространства съдържа специални композиции $X_n \times X_m (n + m = N)$, тогава и останалите три пространства са пространства от същите специални композиции.