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CURVATURE PROPERTIES OF SOME THREE-DIMENSIONAL ALMOST CONTACT B-METRIC MANIFOLDS

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Abstract. The curvature tensor on an arbitrary 3-dimensional Lorentz manifold is expressed by the Ricci tensor and the scalar curvature. The curvature tensor on a 3-dimensional almost contact B-metric manifold belonging to two main classes is studied. The corresponding curvatures are found and the respective geometric characteristics of the considered manifolds are obtained.

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***Key words*:** almost contact manifold, indefinite metric, curvatures.

1. Preliminaries

Let $(M, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional almost contact manifold with B-metric, i.e. (φ, ξ, η) is an almost contact structure and g is a metric on M such that

$$\varphi^2 = -id + \eta \otimes \xi; \quad \eta(\xi) = 1; \quad g(\varphi\cdot, \varphi\cdot) = -g(\cdot, \cdot) + \eta(\cdot)\eta(\cdot). \tag{1.1}$$

Both metrics g and its associated $\tilde{g} : \tilde{g} = g^* + \eta \otimes \eta$ are indefinite metrics of signature $(n, n+1)$ [1], where it is denoted $g^* = g(\cdot, \varphi\cdot)$.

In this paper we study the curvature properties of the almost contact B-metric manifolds of dimension three. This dimension is the lowest possible dimension of these manifolds.

Further, X, Y, Z, W will stand for arbitrary differentiable vector fields on M (i.e. X, Y, Z, $W \in \mathfrak{X}(M)$), and x, y, z, w – arbitrary vectors in the tangential space T_pM to M at some point $p \in M$.

Let $(V, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional vector space with almost contact B-metric structure. Let us denote the subspace $hV := \ker \eta$ of V, and the restrictions of g and φ on hV by the same letters. It is obtained a $2n$-dimensional vector space hV with a complex structure φ and B-metric g. Let $\{e_1, \dots, e_n, \varphi e_1, \dots, \varphi e_n, \xi\}$ be an adapted φ-basis of V, where

$$-g(e_i, e_j) = g(\varphi e_i, \varphi e_j) = \delta_{ij}, g(e_i, \varphi e_j) = 0, \eta(e_i) = 0; i, j \in \{1, \dots, n\}.$$

A decomposition of the class of the almost contact manifolds with B-metric with respect to the tensor

$$F : \ F(X, Y, Z) = g((\nabla_X \varphi)Y, Z)$$

is given in [1], where there are defined eleven basic classes $\mathcal{F}_i$ $(i = 1, \dots, 11)$. The Levi-Civita connection of g is denoted by ∇. The special class $\mathcal{F}_0$: $F = 0$ is contained in each of $\mathcal{F}_i$. The following 1-forms

$$\theta(\cdot) = g^{ij} F(e_i, e_j, \cdot), \quad \theta^*(\cdot) = g^{ij} F(e_i, \varphi e_j, \cdot), \quad \omega(\cdot) = F(\xi, \xi, \cdot)$$

are associated with F, where $\{e_i, \xi\}$ $(i = 1, \dots, 2n)$ is a basis of T_pM, and (g^{ij}) is the inverse matrix of (g_{ij}).

In this paper we consider especially the class $\mathcal{F}_0$ and two of the main classes $\mathcal{F}_4$ and $\mathcal{F}_5$ engendered by the main components of F. Explicit examples of $\mathcal{F}_5$- and $\mathcal{F}_4 \oplus \mathcal{F}_5$-manifolds are given in [1]. Moreover, these classes are analogues of the ones of the known α-Sasakian and $\tilde{\alpha}$-Kenmotsu manifolds in the geometry of the almost contact metric manifolds. The considered classes are determined by the conditions

$$\begin{aligned} &\mathcal{F}_4 : F(X, Y, Z) = -\tfrac{\theta(\xi)}{2n} \{g(\varphi X, \varphi Y)\eta(Z) + g(\varphi X, \varphi Z)\eta(Y)\}, \\ &\mathcal{F}_5 : F(X, Y, Z) = -\tfrac{\theta^*(\xi)}{2n} \{g(X, \varphi Y)\eta(Z) + g(X, \varphi Z)\eta(Y)\}. \end{aligned} \tag{1.2}$$

Let us recall [4] the canonical connection. It is a non-symmetric natural connection D on $(M, \varphi, \xi, \eta, g)$ defined by

$$D_X Y = \nabla_X Y + \frac{1}{2} \{(\nabla_X \varphi)\varphi Y + (\nabla_X \eta)Y.\xi\} - \eta(Y)\nabla_X \xi.$$

The structural tensors $\varphi, \xi, \eta, g, \tilde{g}$ are covariant constants with respect to D.

The curvature tensor R for ∇ is defined as ordinary by $R = [\nabla, \nabla] - \nabla_{[\ ,\ ]}$. The tensor K is the corresponding curvature tensor for D. The corresponding tensor fields of type $(0,4)$ are denoted by the same letters.

Let $\mathcal{R}$ be the set of all curvature-like tensors, i.e. the tensors L having the properties

$$(1.3)\quad L(x,y,z,w) = -L(y,x,z,w) = -L(x,y,w,z), \quad \underset{x,y,z}{\mathfrak{S}} L(x,y,z,w) = 0.$$

In an analogous way of the Ricci tensor ρ and the scalar curvatures τ and $\tilde{\tau}$ of R we denote the following contractions of L:

$$\rho(L)(y,z) = g^{ij} L(e_i, y, z, e_j), \quad \tau(L) = g^{ij}\rho(L)(e_i, e_j), \quad \tilde{\tau}(L) = \tilde{g}^{ij}\rho(L)(e_i, e_j),$$

where $\{e_i\}$ $(i = 1, \dots, 2n+1)$ is a basis of T_pM, and (g^{ij}), $(\tilde{g}^{ij})$ are the inverse matrices of (g_{ij}), $(\tilde{g}_{ij})$, respectively.

As it is known ([3]), in the subclasses

$$\mathcal{F}_4^0 = \{\mathcal{F}_4 \mid \mathrm{d}\theta = 0\} \quad \text{and} \quad \mathcal{F}_5^0 = \{\mathcal{F}_5 \mid \mathrm{d}\theta^* = 0\}$$

the canonical curvature tensor K is a Kähler tensor, i.e. K satisfies the Kähler property

$$K(\cdot,\cdot,\varphi\cdot,\varphi\cdot) = -K(\cdot,\cdot,\cdot,\cdot) \quad \text{and} \quad K \in \mathcal{R}.$$

We use the following curvature-like tensors of type $(0,4)$, which are invariant with respect to the structural group $GL(n,C) \cap O(n,n)) \times I$. The tensor S is a symmetric $(0,2)$-tensor and

$$\begin{aligned}
\psi_1(S)(x,y,z,u) &= g(y,z)S(x,u) - g(x,z)S(y,u) + g(x,u)S(y,z) - g(y,u)S(x,z),\\
\psi_2(S)(x,y,z,u) &= \psi_1(S)(x,y,\varphi z,\varphi u),\\
\psi_3(S)(x,y,z,u) &= -\psi_1(S)(x,y,\varphi z,u) - \psi_1(S)(x,y,z,\varphi u),\\
\psi_4(S)(x,y,z,u) &= \psi_1(S)(x,y,\xi,u)\eta(z) + \psi_1(S)(x,y,z,\xi)\eta(u),\\
\psi_5(S)(x,y,z,u) &= \psi_1(S)(x,y,\xi,\varphi u)\eta(z) + \psi_1(S)(x,y,\varphi z,\xi)\eta(u).
\end{aligned}$$

We put

$$\pi_i = \frac{1}{2}\psi_i(g)\ (i = 1,2,3), \pi_i = \psi_i(g)\ (i = 4,5).$$

It is known [4], that the tensors $\pi_1 - \pi_2 - \pi_4$ and $\pi_3 + \pi_5$ are Kähler tensors.

Let us recall the interconnections between the curvature tensors R and K given in the following

Theorem 1.1 ([2]). *Let* (M,φ,ξ,η,g) *be an* $\mathcal{F}_i^0$*-manifold* $(i=4,5)$. *Then*

- *i=4*
$$K = R + \frac{\xi\theta(\xi)}{2n}\pi_5 + \frac{\theta^2(\xi)}{4n^2}\left\{\pi_2 - \pi_4\right\};$$
- *i=5*
$$K = R + \frac{\xi\theta^*(\xi)}{2n}\pi_4 + \frac{\theta^{*2}(\xi)}{4n^2}\pi_1.$$

A decomposition of $\mathcal{R}$ over (V,φ,ξ,η,g) into 20 mutually orthogonal and invariant factors with respect to the structural group $GL(n,C)\cap O(n,n))\times I$ is obtained in [6]. The partial decomposition $\mathcal{R} = h\mathcal{R}\oplus v\mathcal{R}\oplus w\mathcal{R}$ is received initially and subsequently the decompositions:

$$h\mathcal{R} = \omega_1\oplus\cdots\oplus\omega_{11},\quad v\mathcal{R} = v_1\oplus\cdots\oplus v_5,\quad w\mathcal{R} = w_1\oplus\cdots\oplus w_4.$$

The characteristic conditions of the factors $\omega_i\,(i=1,\ldots,11)$, $v_j\,(j=1,\ldots,5)$, $w_k\,(k=1,\ldots,4)$ are given in [6].

Let us recall [7], an almost contact manifold with B-metric is said to be in one of the classes $w\mathcal{R}$, ω_k, v_r, w_s if R belongs to the corresponding component, where $k=1,\ldots,11$; $r=1,\ldots,5$; $s=1,\ldots,4$.

2. On an arbitrary 3-dimensional Lorentz manifold

Let (M,g) be a 3-dimensional Lorentz manifold, i.e. g is an indefinite metric with signature $(1,2)$.

Theorem 2.1. *Every curvature-like tensor on a 3-dimensional Lorentz manifold has the form*

$$L = \psi_1(\rho(L)) - \frac{\tau(L)}{2}\pi_1.$$

Proof. Let $\{e_1,e_2,e_3\}$ be a pseudo-orthonormal basis on T_pM with respect to g, i.e. for $g_{ij}=g(e_i,e_j)$ we have $-g_{11}=g_{22}=g_{33}=1$, $g_{ij}=0$, $i\neq j$. Then the components of $\rho(L)$ and $\tau(L)$ are: $\rho(L)_{11}=L_{1221}+L_{1331}$, $\rho(L)_{22} = -L_{1221}+L_{2332}$, $\rho(L)_{33} = -L_{1331}+L_{1331}$, $\rho(L)_{12}=L_{3123}$, $\rho(L)_{13}=L_{2132}$, $\rho(L)_{23}=-L_{1231}$, $\tau(L)=-\rho(L)_{11}+\rho(L)_{22}+\rho(L)_{33}$. By direct computations we obtain the above relation between the tensors of type $(0,4)$ for arbitrary vectors in T_pM. □

Corollary 2.2. *The curvature tensor on every 3-dimensional Lorentz manifold has the form*

$$R = \psi_1(\rho) - \frac{\tau}{2}\pi_1. \tag{2.1}$$

3. On a 3-dimensional almost contact B-metric manifold

Let $(M, \varphi, \xi, \eta, g)$ be a 3-dimensional almost contact B-metric manifold. According [1] the class of these manifolds is $\mathcal{F}_1 \oplus \mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_8 \oplus \cdots \oplus \mathcal{F}_{11}$.

From the decomposition of $\mathcal{R}$ it follows that the 3-dimensional almost contact manifold with B-metric cannot belong to the factors $\omega_i\,(i = 1, 2, 3, 4, 9, 10, 11)$, $v_j\,(j = 4, 5)$.

Proposition 3.1. *Every 3-dimensional almost contact B-metric manifold belongs to the class $\omega_5 \oplus v_1 \oplus w\mathcal{R}$.*

Proof. Let $\{e_1, e_2 := \varphi e_1, e_3 := \xi\}$ be a φ-basis of T_pM at every point $p \in M$. For arbitrary $x \in T_pM$ we have the decomposition $x = hx + \eta(x)\xi$, where $hx = x^1 e_1 + x^2 e_2$. We obtain immediately from (2.1) the following components

$$R(hx, hy, hz, hw) = \tfrac{1}{4}\tau\left\{\pi_1 + \pi_2\right\}(hx, hy, hz, hw),$$
$$R(hx, hy, hz, \xi) = \tfrac{1}{2}\left\{\psi_1 + \psi_2\right\}(\rho)(hx, hy, hz, \xi),$$
$$R(\xi, hy, hz, \xi) = y^1 z^1 R_{3113} - g(hy, \varphi hz)R_{3123} + y^2 z^2 R_{3223}.$$

Taking into account the last equalities and the decomposition of $\mathcal{R}$ from [6] we receive $hR \in \omega_5$, $vR \in v_1$, $wR \in w\mathcal{R}$ and consequently $M \in \omega_5 \oplus v_1 \oplus w\mathcal{R}$. $\square$

Using (1.1), the Kähler property of L and Theorem 2.1 we receive immediately the components of $\rho(L)$ and $\tau(L)$ whence we have

Lemma 3.2. *Every Kähler curvature-like tensor L on a 3-dimensional almost contact B-metric manifold is zero.*

It is known [2], the curvature tensor R (resp. K) on an $\mathcal{F}_0$-manifold (resp. $\mathcal{F}_i^0$-manifold, $i = 4, 5$) is a Kähler tensor. Then Lemma 3.2 implies the following two theorems.

Theorem 3.3. *Every 3-dimensional $\mathcal{F}_0$-manifold is flat, i.e.* $R = 0$.

Theorem 3.4. *Every 3-dimensional $\mathcal{F}_i^0$-manifold ($i = 4, 5$) is canonical flat, i.e.* $K = 0$.

Then Theorem 3.4 and Theorem 1.1 imply the following

Proposition 3.5. *Let $(M, \varphi, \xi, \eta, g)$ be a 3-dimensional $\mathcal{F}_i^0$-manifold (i=4,5). Then*

- *i=4*
$$R = -\frac{1}{4}\theta^2(\xi)\{\pi_2 - \pi_4\} - \frac{1}{2}\xi\theta(\xi)\pi_5;$$
- *i=5*
$$R = -\frac{1}{4}\theta^{*2}(\xi)\pi_1 - \frac{1}{2}\xi\theta^*(\xi)\pi_4;$$

Using Corollary 2.2 and Proposition 3.5 we establish the truthfulness of the following

Theorem 3.6. *The curvature tensor, the Ricci tensor and the scalar curvatures on a 3-dimensional $\mathcal{F}_i^0$-manifold ($i = 4, 5$) are respectively:*

- *i=4*
$$R = -\tfrac{1}{2}\tau\{\pi_1 - \pi_3 - 2\pi_4\} - \tfrac{1}{2}\tilde{\tau}\pi_3 = -\tfrac{1}{2}\tau\{\pi_2 - \pi_4 + \pi_5\} + \tfrac{1}{2}\tilde{\tau}\pi_5,$$
$$\rho = -\tfrac{1}{2}\{\tau - \tilde{\tau}\} g^* + \tau\eta \otimes \eta,$$
$$\tau = \rho(\xi, \xi) = \tfrac{1}{2}\theta^2(\xi), \qquad \tilde{\tau} = \tfrac{1}{2}\theta^2(\xi) - \xi\theta(\xi);$$
- *i=5*
$$R = \tfrac{1}{2}\{\tau - 2\tilde{\tau}\}\pi_1 - \tfrac{1}{2}\{\tau - 3\tilde{\tau}\}\pi_4,$$
$$\rho = \tfrac{1}{2}\{\tau - \tilde{\tau}\} g - \tfrac{1}{2}\{\tau - 3\tilde{\tau}\}\eta \otimes \eta,$$
$$\tau = -\tfrac{1}{2}\left\{3\theta^{*2}(\xi) + 4\xi\theta^*(\xi)\right\}, \qquad \tilde{\tau} = \rho(\xi, \xi) = -\tfrac{1}{2}\left\{\theta^{*2}(\xi) + 2\xi\theta^*(\xi)\right\}.$$

According to the decomposition of $\mathcal{R}$ from Theorem 3.6 we receive

Proposition 3.7. *The class of the 3-dimensional $\mathcal{F}_i^0$-manifolds for $i = 4$ and $i = 5$ is $\omega_5 \oplus w_1 \oplus w_2$ for $i = 4$ and $\omega_5 \oplus w_1$ for $i = 5$, respectively.*

It is well known the orthogonal decomposition $V = hV \oplus vV$ of $(V, \varphi, \xi, \eta, g)$ $(\dim V = 2n{+}1)$, where $hV = \{x \in V \mid x = hx = -\varphi^2 x\}$, $vV = \{x \in V| \, x = vx = \eta(x)\xi\}$. Then the restrictions of the B-metrics g and $\tilde{g}$ on hV are $g_h = -g(\varphi\cdot, \varphi\cdot) = g - \eta \otimes \eta$, $\tilde{g}_h = g(\cdot, \varphi\cdot) = g^*$, respectively. On the other side, $\eta \otimes \eta$ is the restriction of the both B-metrics on vV.

Let us introduce the following notions.

Definition 3.8. The $(2n+1)$-dimensional manifold $(M, \varphi, \xi, \eta, g)$ is called **a contact-Einstein manifold** if the Ricci tensor on T_pM has the form $\rho = \alpha g_h + \beta \tilde{g}_h + \gamma \eta \otimes \eta$, where α, β, γ are real constants. A contact-Einstein manifold is called **an h-Einstein manifold**, **a v-Einstein manifold** if $\rho = \alpha g_h + \beta \tilde{g}_h$, $\rho = \gamma \eta \otimes \eta$, respectively. An h-Einstein manifold is called **a φ-Einstein manifold**, **a $*$-Einstein manifold** if $\rho = \alpha g_h$, $\rho = \beta \tilde{g}_h$, respectively.

Note that M is an Einstein manifold (i.e. $\rho = \alpha g$) in the case when $\beta = 0$, $\alpha = \gamma \neq 0$.

Having in mind Theorem 3.6 we give some geometric characteristics of the $\mathcal{F}_i^0$-manifolds $(i = 4, 5)$.

Proposition 3.9. *The 3-dimensional $\mathcal{F}_4^0$-manifolds are not Einstein, φ-Einstein, $*$-Einstein, Ricci-flat manifolds. The 3-dimensional $\mathcal{F}_5^0$-manifolds are not $*$-Einstein, Ricci-flat manifolds.*

Proposition 3.10.

1. *A 3-dimensional $\mathcal{F}_4^0$-manifold is v-Einstein iff* $\theta(\xi)$=const.
2. *A 3-dimensional $\mathcal{F}_5^0$-manifold is*
   (a) *Einstein iff $\theta^*(\xi) = const$;*
   (b) *φ-Einstein iff $2\xi\theta^*(\xi) = -\theta^{*2}(\xi)$.*
   (c) *v-Einstein iff $\xi\theta^*(\xi) = -\theta^{*2}(\xi)$.*

Proposition 3.11.

1. *The scalar curvature τ and the Ricci curvature in the direction of ξ on a 3-dimensional $\mathcal{F}_4^0$-manifold are equal and positive.*

2. *The associated scalar curvature $\tilde{\tau}$ and the Ricci curvature in the direction of ξ on a 3-dimensional $\mathcal{F}_5^0$-manifold are equal.*

The sectional curvature $k(x,y) = \frac{R(x,y,y,x)}{\pi_1(x,y,y,x)}$ with respect to g and R for every nondegenerate section α with a basis $\{x,y\}$ in T_pM, $\dim M = 2n+1$ is known. The special sections in T_pM, $\dim M = 2n+1$: a ξ-section (e.g. $\{\xi, x\}$), a φ-holomorphic section (i.e. $\alpha = \varphi\alpha$) and a totally real section (i.e. $\alpha \perp \varphi\alpha$) are introduced in [5]. Note that the totally real sections in the 3-dimensional case do not exist.

Using Theorem 3.6 we compute the sectional curvatures of a ξ-section and a φ-holomorphic section on a 3-dimensional $\mathcal{F}_i^0$-manifold ($i = 4,5$):

- i=4

$$k(\xi, x) = \frac{\tau}{2}\left\{1 + \frac{g(x,\varphi x)}{g(\varphi x, \varphi x)}\right\} - \frac{\tilde{\tau}}{2}\frac{g(x,\varphi x)}{g(\varphi x,\varphi x)}, \tag{3.1}$$

$$k\left(\varphi x, \varphi^2 x\right) = -\frac{\tau}{2} = -\frac{\theta^2(\xi)}{4};$$

- i=5

$$k(\xi, x) = \frac{\tilde{\tau}}{2} = -\frac{1}{4}\left\{\theta^{*2}(\xi) + 2\xi\theta^*(\xi)\right\}, \tag{3.2}$$

$$k\left(\varphi x, \varphi^2 x\right) = \frac{\tau}{2} - \tilde{\tau} = -\frac{\theta^{*2}(\xi)}{4}.$$

Then according to (3.1) and (3.2) we receive a certain constancy of the special sectional curvatures.

Proposition 3.12. *Every 3-dimensional $\mathcal{F}_i^0$-manifold ($i = 4,5$) has negative point-wise constant φ-holomorphic sectional curvatures. Every 3-dimensional $\mathcal{F}_5^0$-manifold has point-wise sectional curvatures of the ξ-sections.*

Proposition 3.13. *Let (M,φ,ξ,η,g) be a 3-dimensional $\mathcal{F}_i^0$-manifold (i=4,5). Then*

- *for i=4*

  M has positive constant sectional curvatures of the ξ-sections and negative constant φ-holomorphic sectional curvatures iff M is a v-Einstein manifold;

- *for i=5*

  1. *M has negative constant φ-holomorphic sectional curvatures iff M is an Einstein manifold;*

  2. *If M is Einstein then M has constant sectional curvatures of the ξ-sections.*

References

[1] G. Ganchev, V. Mihova, K. Gribachev. *Almost contact manifolds with B-metric*, Math. Balk., 7 (1993), 3-4, 261–276.

[2] M. Manev. *Properties of curvatures tensors on almost contact manifolds with B-metric*, Proc. of Jubilee Sci. Session of The V. Levsky Higher Military School, V. Tarnovo, 27 (1993), 221–227.

[3] M. Manev, K. Gribachev. *Contactly conformal transformations of almost contact manifolds with B-metric*, Serdica - Bulg. Math. Publ., 19 (1993), 287–299.

[4] M. Manev, K. Gribachev. *Contactly invariant tensors on almost contact manifolds with B-metric*, Serdica - Bulg. Math. Publ., 20 (1994), 133–147.

[5] G. Nakova and K. Gribachev. *Submanifolds of some almost contact manifolds with B-metric with codimension two. I*, Math. Balkanica (N.S.), 12 (1998), 1-2, 93–108.

[6] G. Nakova. *Curvature tensors on almost contact manifolds with B-metric*, Perspectives of Complex Analysis, Differential Geometry and Mathematical Physics, eds. S. Dimiev and K. Sekigawa, World Scientific, Singapore. (to appear)

[7] G. Nakova. *Curvature tensors in the basic classes of real isotropic hypersurfaces of a Kaehler manifold with B-metric*, Perspectives of Complex Analysis, Differential Geometry and Mathematical Physics, eds. S. Dimiev and K. Sekigawa, World Scientific, Singapore. (to appear)

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КРИВИННИ СВОЙСТВА НА НЯКОИ ТРИМЕРНО ПОЧТИ КОНТАКТНИ В-МЕТРИЧНИ МНОГООБРАЗИЯ

Манчо Манев, Галя Накова

Резюме. В тази статия се изразява кривинният тензор върху произволно лоренцово многообразие чрез тензора на Ричи и скаларната кривина. Разгледани са тримерни почти контактни B-метрични многообразия, принадлежащи на два главни класа. Изучен е кривинния тензор върху тези многообразия. Намерени са съответните кривини и са получени свързаните с тях геометрични характеристики на разгледаните многообразия.